

ENGINEERING MECHANICS

CHAPTER 2: COPLANAR CONCURRENT FORCES

Lecture 5:

HOME WORK: Find the magnitude of the two forces, such that if they act at right angles, their resultant is $\sqrt{10}$ N. But, if they act at 60° , their resultant is $\sqrt{13}$ N. (ANS- 3N and 1N)

Principle of Resolution of forces (Or Principle of Resolved parts):

It states, “The algebraic sum of the resolved parts of a number of forces, in a given direction is equal to the resolved parts of their resultant in the same direction.”

NOTE- The force which is split into two parts is called the resolved force and the parts are called component forces or resolute.

Method of resolution of forces:

The magnitude and direction of the resultant force can also be found out using the method of resolution of forces as discussed below:

- Resolve all the forces horizontally and find the algebraic sum of all the horizontal components (i.e., $\sum H$).
- Resolve all the forces vertically and find the algebraic sum of all the vertical components (i.e., $\sum V$).
- The resultant R of the given forces will be given by the equation : $R = \sqrt{(\sum H)^2 + (\sum V)^2}$
- The resultant force will be inclined at an angle θ , with the horizontal, such that

$$\tan \theta = \frac{\sum V}{\sum H}$$

Notes : The value of the angle θ will vary depending upon the values of $\sum V$ and $\sum H$ as discussed below :

1. When $\sum V$ is +ve, the resultant makes an angle between 0° and 180° . But when $\sum V$ is –ve, the resultant makes an angle between 180° and 360° .
2. When $\sum H$ is +ve, the resultant makes an angle between 0° to 90° or 270° to 360° . But when $\sum H$ is –ve, the resultant makes an angle between 90° to 270° .

Example 2.6. A system of forces are acting at the corners of a rectangular block as shown in Fig. 2.4.

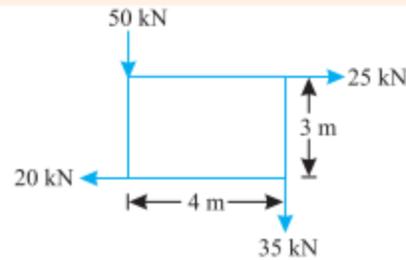


Fig. 2.4.

Determine the magnitude and direction of the resultant force.

Solution. Given : System of forces

Magnitude of the resultant force

Resolving forces horizontally,

$$\sum H = 25 - 20 = 5 \text{ kN}$$

and now resolving the forces vertically

$$\sum V = (-50) + (-35) = -85 \text{ kN}$$

\therefore Magnitude of the resultant force

$$R = \sqrt{(\sum H)^2 + (\sum V)^2} = \sqrt{(5)^2 + (-85)^2} = 85.15 \text{ kN} \quad \text{Ans.}$$

Direction of the resultant force

Let θ = Angle which the resultant force makes with the horizontal.

We know that

$$\tan \theta = \frac{\sum V}{\sum H} = \frac{-85}{5} = -17 \quad \text{or} \quad \theta = 86.6^\circ$$

Since $\sum H$ is positive and $\sum V$ is negative, therefore resultant lies between 270° and 360° . Thus actual angle of the resultant force

$$= 360^\circ - 86.6^\circ = 273.4^\circ \quad \text{Ans.}$$

Example 2.7 The forces 20 N, 30 N, 40 N, 50 N and 60 N are acting at one of the angular points of a regular hexagon, towards the other five angular points, taken in order. Find the magnitude and direction of the resultant force.

Solution. The system of given forces is shown in Fig. 2.5

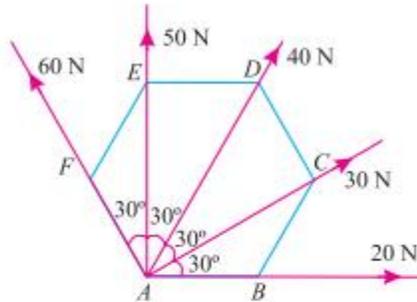


Fig. 2.5.

Magnitude of the resultant force

Resolving all the forces horizontally (*i.e.*, along AB),

$$\begin{aligned}\sum H &= 20 \cos 0^\circ + 30 \cos 30^\circ + 40 \cos 60^\circ + 50 \cos 90^\circ + 60 \cos 120^\circ \text{ N} \\ &= (20 \times 1) + (30 \times 0.866) + (40 \times 0.5) + (50 \times 0) + 60 (-0.5) \text{ N} \\ &= 36.0 \text{ N} \quad \dots(i)\end{aligned}$$

and now resolving the all forces vertically (*i.e.*, at right angles to AB),

$$\begin{aligned}\sum V &= 20 \sin 0^\circ + 30 \sin 30^\circ + 40 \sin 60^\circ + 50 \sin 90^\circ + 60 \sin 120^\circ \text{ N} \\ &= (20 \times 0) + (30 \times 0.5) + (40 \times 0.866) + (50 \times 1) + (60 \times 0.866) \text{ N} \\ &= 151.6 \text{ N} \quad \dots(ii)\end{aligned}$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\sum H)^2 + (\sum V)^2} = \sqrt{(36.0)^2 + (151.6)^2} = 155.8 \text{ N} \quad \text{Ans.}$$

Direction of the resultant force

Let θ = Angle, which the resultant force makes with the horizontal (*i.e.*, AB).

We know that

$$\tan \theta = \frac{\sum V}{\sum H} = \frac{151.6}{36.0} = 4.211 \quad \text{or} \quad \theta = 76.6^\circ \quad \text{Ans.}$$

HOMEWORK: Find the magnitude and direction of the resultant of the concurrent forces of 8N, 12N, 15N and 20N making angles $30^\circ, 70^\circ, 120^\circ, 150^\circ$ respectively from a fixed line. (2021-5 marks, same as the above sum)