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ENGINEERING MECHANICS

CHAPTER 10: SIMPLE LIFTING MACHINES

Lecture 3:

10.3 Simple lifting Machines –simple Wheel and axle, differential wheel and axle and screw jack(simple) problems.

Simple wheel and axle:

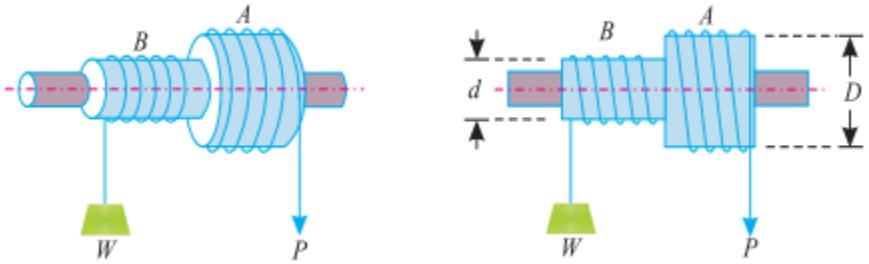

Figure: Simple wheel and axle

Figure shows a simple wheel and axle, in which the wheel A and axle B are keyed to the same shaft. The shaft is mounted on ball bearings in order to reduce the frictional resistance to a minimum. A string is wound round the axle B, which carries the load to be lifted. A second string is wound round the wheel A in the opposite direction to that of the string on B.

Let D = Diameter of effort wheel,
d = Diameter of the load axle,
W = Load lifted, and
P = Effort applied to lift the load.

One end of the string is fixed to the wheel, while the other is free and the effort is applied to this end. Since the two strings are wound in opposite directions, therefore a downward motion of the effort (P) will raise the load (W). Since the wheel as well as the axle are keyed to the same shaft, therefore when the wheel rotates through one revolution, the axle will also rotate through one revolution.

We know that displacement of the effort in one revolution of effort wheel A = πD ...(i)

and displacement of the load in one revolution = πd ...(ii)

$$\therefore \quad \text{V.R.} = \frac{\text{Distance moved by the effort}}{\text{Distance moved by the load}} = \frac{\pi D}{\pi d} = \frac{D}{d}$$

Now $$\text{M.A.} = \frac{\text{Load lifted}}{\text{Effort applied}} = \frac{W}{P}$$ ..as usual

and efficiency $$\eta = \frac{\text{M.A}}{\text{V.R.}}$$ ...as usual

Example 11.1 *A simple wheel and axle has wheel and axle of diameters of 300 mm and 30 mm respectively. What is the efficiency of the machine, if it can lift a load of 900 N by an effort of 100 N.*

Solution. Given: Diameter of wheel (D) = 300 mm; Diameter of axle (d) = 30 mm; Load lifted by the machine (W) = 900 N and effort applied to lift the load (P) = 100 N

We know that velocity ratio of the simple wheel and axle,

$$\text{V.R.} = \frac{D}{d} = \frac{300}{30} = 10$$

and mechanical advantage $$\text{M.A.} = \frac{W}{P} = \frac{900}{100} = 9$$

$$\therefore \quad \text{Efficiency, } \eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{9}{10} = 0.9 = 90\% \quad \textbf{Ans.}$$

Example 11.2 *A drum weighing 60 N and holding 420N of water is to be raised from a well by means of wheel and axle. The axle is 100 mm diameter and the wheel is 500 mm diameter. If a force of 120 N has to be applied to the wheel, find (i) mechanical advantage, (ii) velocity ratio and (iii) efficiency of the machine.*

Solution. Given: Total load to be lifted (W) = 60 + 420 = 480 N; Diameter of the load axle (d) = 100 mm; Diameter of effort wheel (D) = 500 mm and effort (P) = 120 N.

Mechanical advantage

We know that mechanical advantage

$$\text{M.A.} = \frac{W}{P} = \frac{480}{120} = 4 \quad \textbf{Ans.}$$

Velocity ratio

We know that velocity ratio

$$\text{V.R.} = \frac{D}{d} = \frac{500}{100} = 5 \quad \textbf{Ans.} \qquad ...(ii)$$

Efficiency of the machine

We also know that efficiency of the machine,

$$\eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{4}{5} = 0.8 = 80\% \quad \textbf{Ans.}$$

Note : If we consider weight of the water only (*i.e.*, neglecting weight of the drum) then

$$\text{M.A.} = \frac{420}{120} = 3.5 \quad \textbf{Ans.}$$

and efficiency $$\eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{3.5}{5} = 0.7 = 70\% \quad \textbf{Ans.}$$

Differential wheel and axle:

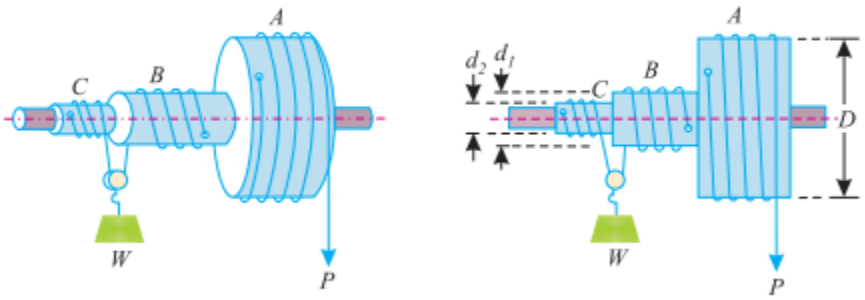

Figure: Differential wheel and axle

It is an improved form of simple wheel and axle, in which the velocity ratio is intensified with the help of load axle. Figure shows a differential wheel and axle. In this case, the load axle BC is made up of two parts of different diameters. Like simple wheel and axle, the wheel A, and the axles B and C are keyed to the same shaft, which is mounted on ball bearings in order to reduce the frictional resistance to a minimum.

The effort string is wound round the wheel A. Another string is wound round the axle B, which after passing round the pulley (to which the weight W is attached) is wound round the axle C in opposite direction to that of the axle B; care being taken to wind the string on the wheel A and axle C in the same direction. As a result of this, when the string unwinds from the wheel A, the other string also unwinds from the axle C. But it winds on the axle B as shown in figure.

Let D = Diameter of the effort wheel A,
d_1= Diameter of the axle B,
d_2 = Diameter of the axle C,
W = Weight lifted by the machine, and
P = Effort applied to lift the weight.

We know that displacement of the effort in one revolution of effort wheel A = πD ...(i)

∴ Length of string, which will wound on axle B in one revolution = πd_1

and length of string, which will unwound from axle C in one revolution = $\pi\ d_2$

∴ Length of string which will wound in one revolution = $\pi\ d_1 - \pi\ d_2 = \pi\ (d_1 - d_2)$

and displacement of weight $= \frac{1}{2} \times \pi(d_1 - d_2) = \frac{\pi}{2}(d_1 - d_2)$ ...(*ii*)

$\therefore$ $\text{V.R.} = \frac{\text{Distance moved by the effort}}{\text{Distance moved by the load}} = \frac{\pi D}{\frac{\pi}{2}(d_1 - d_2)} = \frac{2D}{d_1 - d_2}$

Now $\text{M.A.} = \frac{W}{P}$ ...as usual

and efficiency, $\eta = \frac{\text{M.A.}}{\text{V.R.}}$ ...as usual

Example 11.3 *The larger and smaller diameters of a differential wheel and axle are 80 mm and 70 mm respectively. The effort is applied to the wheel of diameter 250 mm. What is the velocity ratio?*

Find efficiency and frictional effort lost, when a load of 1050 N is lifted by an effort of 25 N.

Solution. Given: Larger diameter of wheel (d_1) = 80 mm; Smaller diameter of wheel (d_2) = 70 mm; Diameter of the effort wheel (D) = 250 mm; Load lifted (W) = 1050 N and effort (P) = 25 N.

Velocity ratio

We know that velocity ratio

$$\text{V.R.} = \frac{2D}{d_1 - d_2} = \frac{2 \times 250}{80 - 70} = 48 \quad \textbf{Ans.} \qquad ...(i)$$

Efficiency

We know that mechanical advantage

$$\text{M.A.} = \frac{W}{P} = \frac{1050}{25} = 42 \qquad ...(ii)$$

and efficiency, $\eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{42}{50} = 0.84 = 84\%$ **Ans.**

Frictional effort lost

We also know that frictional effort lost,

$$F_{(\text{effort})} = P - \frac{\text{W}}{\text{V.R.}} = 25 - \frac{1050}{50} = 4 \text{ N} \quad \textbf{Ans.}$$

Example 11.1. *With a differential wheel and axle, an effort of 6 N raised a load of 60 N. If the efficiency at this load is 80%, find the velocity ratio of the machine.*

If the diameter of the effort wheel is 300 mm, determine the difference betwen the diameters of the axles. If the sum of the diameters of the axles is 280 mm, determine the diameter of each axle.

Solution: Given: Effort (P) = 6 N; Load raised (W) = 60 N; Efficiency (η) = 80% = 0.8; Diameter of effort wheel (D) = 300 mm and sum of the diameters of axles ($d_1 + d_2$) = 280 mm.

Velocity ratio of the machine

We know that mechanical advantage

$$\text{M.A.} = \frac{W}{P} = \frac{60}{6} = 10$$

and efficiency,

$$0.8 = \frac{\text{M.A.}}{\text{V.R.}} = \frac{10}{\text{V.R.}}$$

$$\therefore \quad \text{V.R.} = \frac{10}{0.8} = 12.5 \qquad \textbf{Ans.}$$

Difference between the diameters of the axles

We know that velocity ratio of a differential wheel and axle,

$$12.5 = \frac{2D}{d_1 - d_2} = \frac{2 \times 300}{d_1 - d_2} = \frac{600}{d_1 - d_2}$$

$$\therefore \quad (d_1 - d_2) = \frac{600}{12.5} = 48 \qquad \textbf{Ans.}$$

Diameter of each axle

Solving $(d_1 - d_2) = 48$ and $(d_1 + d_2) = 280$ simultaneously, we get

$$d_1 = 164 \text{ mm and } d_2 = 116 \text{ mm} \qquad \textbf{Ans.}$$

Example 11.5 *In a differential wheel and axle, the diameter of the effort wheel is 400 mm. The radii of the axles are 150 mm and 100 mm respectively. The diameter of the rope is 10 mm.*

Find the load which can be lifted by an effort of 25 N assuming the efficiency of the machine to be 84%.

Solution. Given: Diameter of effort wheel = 480 mm; Radii of axles = 150 mm and 100 mm or diameter of axles = 300 mm and 200 mm; Diameter of rope = 10 mm; Effort (P) = 25 N and efficiency (η) = 84% = 0.84.

Let W = Load that can be lifted by the machine.

We know that effective diameter of the effort wheel,

$$D = 400 + 10 = 410 \text{ mm}$$

and effective diameters of axles,

$$d_1 = 300 + 10 = 310 \text{ mm and } d_2 = 200 + 10 = 210 \text{ mm}$$

We also know that velocity ratio of a differential wheel and axle,

$$\text{V.R.} = \frac{2D}{d_1 - d_2} = \frac{2 \times 410}{310 - 210} = 8.2$$

and

$$\text{M.A.} = \frac{W}{P} = \frac{W}{25}$$

We also know that efficiency,

$$84 = \frac{\text{M.A.}}{\text{V.R.}} = \frac{\frac{W}{25}}{8.2} = \frac{W}{205}$$

or $$W = 0.84 \times 205 = 172.2 \text{ N}$$ **Ans.**

Simple screw jack:

It consists of a screw, fitted in a nut, which forms the body of the jack. The principle, on which a screw jack works, is similar to that of an inclined plane.

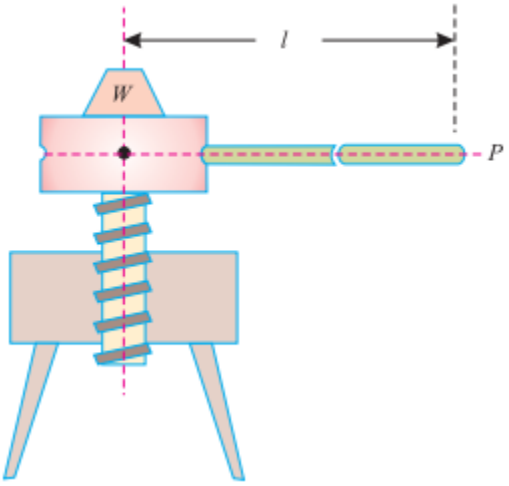

Figure: Simple Screw Jack

Figure shows a simple screw jack, which is rotated by the application of an effort at the end of the lever, for lifting the load. Now considering a single threaded simple screw jack,

Let l = Length of the effort arm,
p = Pitch of the screw,
W = Load lifted, and
P = Effort applied to lift the load at the end of the lever.

We know that distance moved by the effort in one revolution of screw, = 2πl ...(i)

and distance moved by the load = p ...(ii)

$$\therefore \quad \text{Velocity ratio} = \frac{\text{Distance moved by the effort}}{\text{Distance moved by the load}} = \frac{2\pi l}{p} \qquad ...(iii)$$

Now $\quad \text{M.A.} = \frac{W}{P}$ ...as usual

and efficeincy, $\quad \eta = \frac{\text{M.A.}}{\text{V.R.}}$ ...as usual

Note: The value of P *i.e.*, the effort applied may also found out by the relation :

$$*P = W \tan (\alpha + \phi)$$

where W = Load lifted

$$\tan \alpha = \frac{p}{\pi d}$$

and $\tan \phi = \mu$ = Coefficient of friction.

Example *A screw jack has a thread of 10 mm pitch. What effort applied at the end of a handle 400 mm long will be required to lift a load of 2 kN, if the efficiency at this load is 45%.*

Solution. Given: Pitch of thread (p) = 10 mm; Length of the handle (l) = 400 mm; Load lifted (W) = 2 kN = 2000 N and efficiency (n) = 45% = 0.45.

Let P = Effort required to lift the load.

We know that velocity ratio

$$\text{V.R.} = \frac{2\pi l}{p} = \frac{2\pi \times 400}{10} = 251.3$$

and

$$\text{M.A.} = \frac{W}{P} = \frac{2000}{P}$$

We also know that efficiency,

$$0.45 = \frac{\text{M.A.}}{\text{V.R.}} = \frac{\frac{2000}{P}}{251.3} = \frac{7.96}{P}$$

$$P = \frac{7.96}{0.45} = 17.7 \text{ N} \quad \textbf{Ans.}$$

OBJECTIVE TYPE QUESTIONS

1. The velocity ratio of a simple wheel and axle with D and d as the diameters of effort wheel and load axle is :

(*a*) $D + d$ (*b*) $D - d$ (*c*) $D \times d$ (*d*) D/d

2. The velocity ratio of a differential wheel and axle with D as the diameter effort wheel and d_1 and d_2 as the diameters of larger and smaller axles respectively is

(*a*) $\frac{2D}{d_1 + d_2}$ (*b*) $\frac{2D}{d_1 - d_2}$ (*c*) $\frac{D}{d_1 + d_2}$ (*d*) $\frac{D}{d_1 - d_2}$

3.

In a simple screw jack, with (l) as the length of the effort wheel and (p) as pitch of the screw, its velocity ratio is

(*a*) $\frac{2\pi l}{p}$ (*b*) $\frac{\pi l}{2p}$ (*c*) $\frac{2\pi p}{l}$ (*d*) $\frac{\pi p}{2l}$