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Lecturer, Mechanical Engineering

ENGINEERING MECHANICS

CHAPTER 2: COPLANAR CONCURRENT FORCES

Lecture 4:

2.4 Resultant for system of forces, Triangle Law of forces, Parallelogram and Polygon Law of Forces, condition of equilibrium of coplanar current forces, Bow's notation, Lami's theorem, Analytical and Graphical methods of Problem solution.

Resultant for system of forces: The resultant force for a system of forces can be found out using the following laws:
1) Parallelogram Law of Forces
2) Triangle Law of Forces
3) Polygon Law of Forces

Methods used to find the resultant force:
1) Analytical Methods

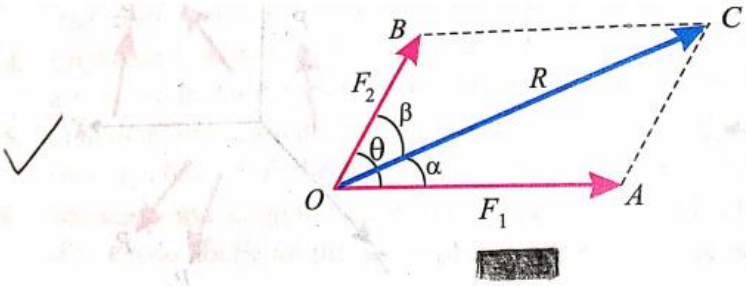
(a) Parallelogram Law of Forces
(b) Method of Resolution Of Forces

2) Graphical or Vector Methods

Parallelogram Law of Forces:

It states that, "If two forces, acting simultaneously on a particle, be represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant may be represented in magnitude and direction by the diagonal of the parallelogram, which passes through their point of intersection."

We consider two forces F_1 and F_2 acting at same point O as shown below.



Let F_1 and F_2 = Forces whose resultant is required to be found out

R = Resultant of forces F_1 and F_2

θ = Angle between the forces F_1 and F_2

α = Angle which the resultant force R makes with one force F_1

β = Angle which the resultant force R makes with the other force F_2.

Now according to the Parallelogram Law of Forces, the Resulant (R) is given as,

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

The Angles α and β can be determined from the relations,

$$\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2\cos\theta}$$

and

$$\tan\beta = \frac{F_1 \sin\theta}{F_2 + F_1\cos\theta}$$

Important Results:

1. If $\theta = 0$ *i.e.*, when the forces act along the same line, then
$$R = F_1 + F_2$$ ...(Since $\cos 0° = 1$)

2. If $\theta = 90°$ *i.e.*, when the forces act at right angle, then
$$R = \sqrt{F_1^2 + F_2^2}$$ ...(Since $\cos 90° = 0$)

3. If $\theta = 180°$ *i.e.*, when the forces act along the same straight line but in opposite directions, then $R = F_1 - F_2$ ...(Since $\cos 180° = -1$)
In this case, the resultant force will act in the direction of the greater force.

4. If the two forces are equal *i.e.*, when $F_1 = F_2$
then $$R = \sqrt{F^2 + F^2 + 2F^2 \cos\theta} = \sqrt{2F^2 (1 + \cos\theta)}$$
$$= \sqrt{2F^2 \times 2\cos^2\left(\frac{\theta}{2}\right)} \quad \ldots\left[\because 1 + \cos\theta = 2\cos^2\left(\frac{\theta}{2}\right)\right]$$
$$= \sqrt{4F^2 \cos^2\left(\frac{\theta}{2}\right)} = 2F\cos\left(\frac{\theta}{2}\right)$$

Example 1. *Two forces of 100 N and 150 N are acting simultaneously at a point. What is the resultant of these two forces, if the angle between them is 45°?*

Solution. Given : First force $(F_1) = 100$ N; Second force $(F_2) = 150$ N and angle between F_1 and F_2 $(\theta) = 45°$.

We know that, the Resultant Force,

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$
$$= \sqrt{(100)^2 + (150)^2 + 2\times100\times150\cos 45°}\ \text{N}$$
$$= \sqrt{10\,000 + 22\,500 + (30\,000\times0.707)}\ \text{N}$$
$$= 232\ \text{N} \quad \textbf{Ans.}$$

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Example 2. *Two forces act at an angle of 120°. The bigger force is of 40 N and the resultant is perpendicular to the smaller one. Find the smaller force.*

Solution. Given : Angle between the forces $\angle AOC = 120°$, Bigger force $(F_1) = 40$ N and angle between the resultant and F_2 $(\angle BOC) = 90°$;

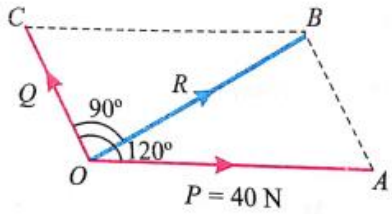

Fig. 7.

Let F_2 = Smaller force in N

From the geometry of the figure, we find that $\angle AOB$,

$$\alpha = 120° - 90° = 30°$$

We know that

$$\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta}$$

or $$\tan 30° = \frac{F_2 \sin 120°}{40 + F_2 \cos 120°} = \frac{F_2 \sin 60°}{40 + F_2 (-\cos 60°)}$$

or $$0.577 = \frac{F_2 \times 0.866}{40 - F_2 \times 0.5} = \frac{0.866\, F_2}{40 - 0.5\, F_2}$$

or $$40 - 0.5 F_2 = \frac{0.866 F_2}{0.577} = 1.5\, F_2$$

or $$2F_2 = 40 \quad \text{or} \quad F_2 = 20$$ **Ans.**