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Lecturer, Mechanical Engineering

ENGINEERING MECHANICS

CHAPTER 2: COPLANAR CONCURRENT FORCES

Lecture 6:

Bow's Notation: It is a method of representing a force by writing two capital letters one on either side of the force. In the figure below, the force P_1 (20 kN) is represented by AB and the force P_2 (10 kN) by CD.

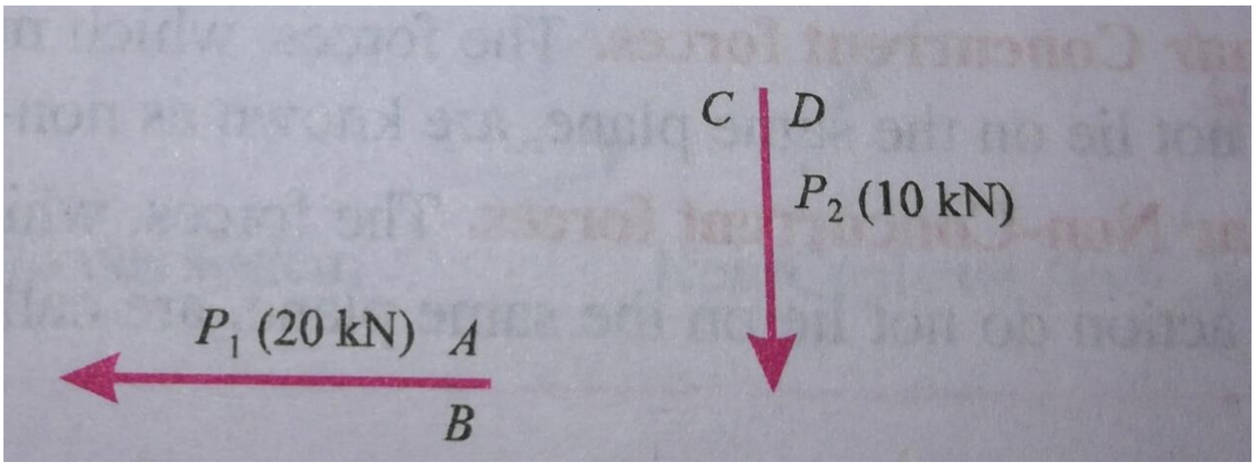

Fig: Bow's Notation of Force

Triangle Law of Forces: It states, "If two forces acting simultaneously on a particle, be represented in magnitude and direction by the two sides of a triangle, taken in order ; their resultant may be represented in magnitude and direction by the third side of the triangle, taken in opposite order."

Polygon Law of Forces:

It is an extension of Triangle Law of Forces for more than two forces, which states, "If a number of forces acting simultaneously on a particle, be represented in magnitude and direction, by the sides of a polygon taken in order ; then the resultant of all these forces may be represented, in magnitude and direction, by the closing side of the polygon, taken in opposite order."

Example 2.5. *A triangle ABC has its side AB = 40 mm along positive x-axis and side BC = 30 mm along positive y-axis. Three forces of 40 N, 50 N and 30 N act along the sides AB, BC and CA respectively. Determine magnitude of the resultant of such a system of forces.*

Solution. The system of given forces is shown in Fig. 2.3.

From the geometry of the figure, we find that the triangle *ABC* is a right angled triangle, in which the *side *AC* = 50 mm. Therefore

$$\sin\theta = \frac{30}{50} = 0.6$$

and

$$\cos\theta = \frac{40}{50} = 0.8$$

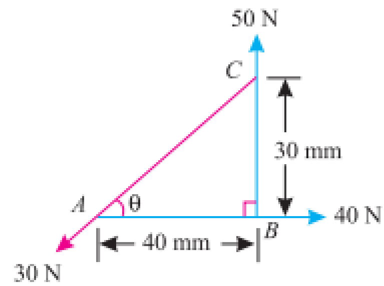

Fig. 2.3.

Resolving all the forces horizontally (*i.e.*, along *AB*),

$$\Sigma H = 40 - 30\cos\theta$$

$$= 40 - (30 \times 0.8) = 16 \text{ N}$$

and now resolving all the forces vertically (*i.e.*, along *BC*)

$$\Sigma V = 50 - 30\sin\theta$$

$$= 50 - (30 \times 0.6) = 32 \text{ N}$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(16)^2 + (32)^2} = 35.8 \text{ N} \quad \textbf{Ans.}$$

Example 2.9. *A horizontal line PQRS is 12 m long, where PQ = QR = RS = 4 m. Forces of 1000 N, 1500 N, 1000 N and 500 N act at P, Q, R and S respectively with downward direction. The lines of action of these forces make angles of 90°, 60°, 45° and 30° respectively with PS. Find the magnitude, direction and position of the resultant force.*

Solution. The system of the given forces is shown in Fig. 2.7

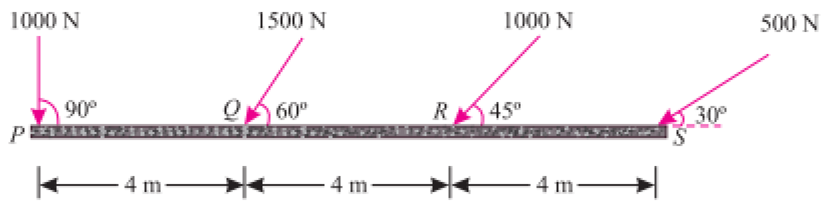

Fig. 2.7.

Magnitude of the resultant force

Resolving all the forces horizontally,

$$\Sigma H = 1000 \cos 90° + 1500 \cos 60° + 1000 \cos 45° + 500 \cos 30° \text{ N}$$

$$= (1000 \times 0) + (1500 \times 0.5) + (1000 \times 0.707) + (500 \times 0.866) \text{ N}$$

$$= 1890 \text{ N} \qquad \text{...}(i)$$

and now resolving all the forces vertically,

$$\Sigma V = 1000 \sin 90° + 1500 \sin 60° + 1000 \sin 45° + 500 \sin 30° \text{ N}$$

$$= (1000 \times 1.0) + (1500 \times 0.866) + (1000 \times 0.707) + (500 \times 0.5) \text{ N}$$

$$= 3256 \text{ N} \qquad \text{...}(ii)$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(1890)^2 + (3256)^2} = 3765 \text{ N} \quad \textbf{Ans.}$$

Direction of the resultant force

Let θ = Angle, which the resultant force makes with *PS*.

$$\therefore \qquad \tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{3256}{1890} = 1.722 \quad \text{or} \quad \theta = 59.8° \quad \textbf{Ans.}$$

Position of the resultant force

Let x = Distance between *P* and the line of action of the resultant force.

Now taking **moments of the resultant force and the vertical components of the applied forces about *P*, and equating the same,**

$$3765\,x = (1000 \times 0) + (1500 \times 0.866)\,4 + (1000 \times 0.707)\,8 + (500 \times 0.5)\,12$$

$$= 13\,852$$

$$\text{or} \qquad x = \frac{13\,852}{3765} = 3.68 \text{ m} \quad \textbf{Ans.}$$