MIMLI DAS

Lecturer, Mechanical Engineering

ENGINEERING MECHANICS

CHAPTER 2: COPLANAR CONCURRENT FORCES

Lecture 8:

Conditions and Equations of Static Equilibrium for coplanar concurrent forces:

If a number of coplanar concurrent forces act on a body, the body is said to be at rest or in equilibrium, if the resultant of all the forces acting on the body is zero. Or in other words, the horizontal component of all the forces ($\sum H$) and vertical component of all the forces ($\sum V$) must be zero. Mathematically , conditions for equilibrium for coplanar concurrent forces are:

$$\sum H=0 \text{ and } \sum V=0 \quad \ldots\ldots\ldots\ldots\ldots\ldots(i)$$

Thus, if a body is in equilibrium under the action of a number of coplanar concurrent forces, the equation (i) must be satisfied.

Lami's Theorem: It states, "If three coplanar forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle between the other two."

Mathematically $\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$ where, P, Q, and R are three forces and α, β, γ are the angles as shown in the figure below.

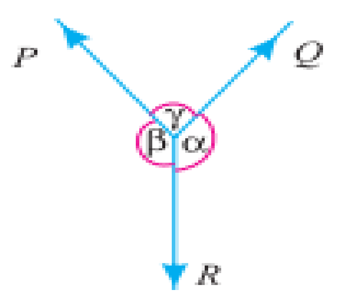

Fig: Lami's Theorem

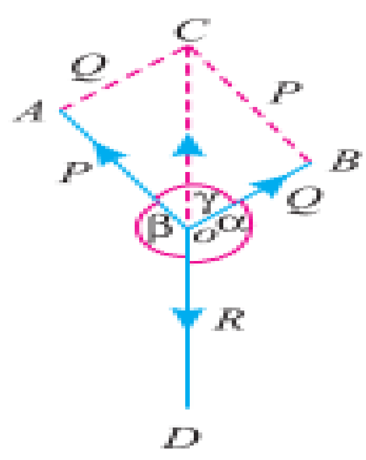

Fig: Proof of Lami's Theorem

Proof: We consider three coplanar forces P, Q, and R acting at a point O. Let the opposite angles to three forces be α , β and γ as shown in Figure above. Now let us complete the parallelogram

OACB with OA and OB as adjacent sides as shown in the figure. We know that the resultant of two forces P and Q will be given by the diagonal OC both in magnitude and direction of the parallelogram OACB. Since these forces are in equilibrium, therefore the resultant of the forces P and Q must be in line with OD and equal to R, but in opposite direction.

From the geometry of the figure, we find BC = P and AC = Q

$\therefore \angle AOC = (180° - \beta)$ and $\angle ACO = \angle BOC = (180° - \alpha)$

$\therefore \angle CAO = 180° - (\angle AOC + \angle ACO)$

$= 180° - [(180° - \beta) + (180° - \alpha)]$

$= 180° - 180° + \beta - 180° + \alpha$

$= \alpha + \beta - 180°$

But $\alpha + \beta + \gamma = 360°$,

Subtracting 180° from both sides of the above equation,

$(\alpha + \beta - 180°) + \gamma = 360° - 180° = 180°$

or $\angle CAO = 180° - \gamma$

We know that in triangle AOC,

$$\frac{OA}{\sin{<}ACO} = \frac{AC}{\sin{<}AOC} = \frac{OC}{\sin{<}CAO}$$

Or, $$\frac{OA}{\sin(180° - \alpha)} = \frac{AC}{\sin(180° - \beta)} = \frac{OC}{\sin(180° - \gamma)}$$

Or, $$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$ ……..[since $\sin(180° - \theta) = \sin\theta$]

Example 5.1. *An electric light fixture weighting 15 N hangs from a point C, by two strings AC and BC. The string AC is inclined at 60° to the horizontal and BC at 45° to the horizontal as shown in Fig. 5.3*

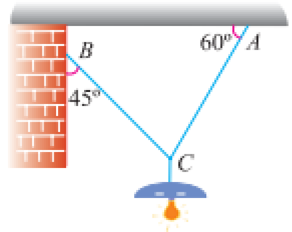

Fig. 5.3.

Using Lami's theorem, or otherwise, determine the forces in the strings AC and BC.

Solution. Given : Weight at C = 15 N

Let T_{AC} = Force in the string AC, and

T_{BC} = Force in the string BC.

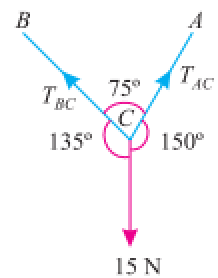

Fig. 5.4.

The system of forces is shown in Fig. 5.4. From the geometry of the figure, we find that angle between T_{AC} and 15 N is 150° and angle between T_{BC} and 15 N is 135°.

$$\therefore \quad \angle ACB = 180° - (45° + 60°) = 75°$$

Applying Lami's equation at C,

$$\frac{15}{\sin 75°} = \frac{T_{AC}}{\sin 135°} = \frac{T_{BC}}{\sin 150°}$$

or

$$\frac{15}{\sin 75°} = \frac{T_{AC}}{\sin 45°} = \frac{T_{BC}}{\sin 30°}$$

$$\therefore \quad T_{AC} = \frac{15 \sin 45°}{\sin 75°} = \frac{15 \times 0.707}{0.9659} = 10.98 \text{ N} \quad \textbf{Ans.}$$

and

$$T_{BC} = \frac{15 \sin 30°}{\sin 75°} = \frac{15 \times 0.5}{0.9659} = 7.76 \text{ N} \quad \textbf{Ans.}$$

OBJECTIVE TYPE QUESTIONS

1. According to Lami's Theorem, the three forces
   (*a*) Must be equal.
   (*b*) Must be at 120° to each other.
   (*c*) Must be both of above.
   (*d*) May not be any of the two.
2. The Lami's Theorem is applicable only for
   (*a*) Coplaner forces (*b*) Concurrent forces
   (*c*) Coplaner and concurrent forces (*d*) Any type of forces
3. If a body is in equilibrium. We may conclude that
   (*a*) No force is acting on the body
   (*b*) The resultant of all the forces acting on it is zero.
   (*c*) The moments of the forces about any point is zero.
   (*d*) Both (*b*) and (*c*)
4. If the sum of all the forces acting on a body is zero, then the body may be in equilibrium provided the forces are
   (*a*) Concurrent (*b*) Parallel
   (*c*) Like parallel (*d*) Unlike parallel
5. A body is said to be in equilibrium, if it has no linear motion.
   (*a*) True (*b*) False
6. Lami's Theorem can not be applied in case of concurrent forces
   (*a*) Agree (*b*) Disagree

ANSWERS

1. (*d*) 2. (*a*) 3. (*d*) 4. (*a*) 5. (*b*) 6. (*b*)

Example 5.4. *Two equal heavy spheres of 50 mm radius are in equilibrium within a smooth cup of 150 mm radius. Show that the reaction between the cup of one sphere is double than that between the two spheres.*

Solution. Given : Radius of spheres = 50 mm and radius of the cup = 150 mm.

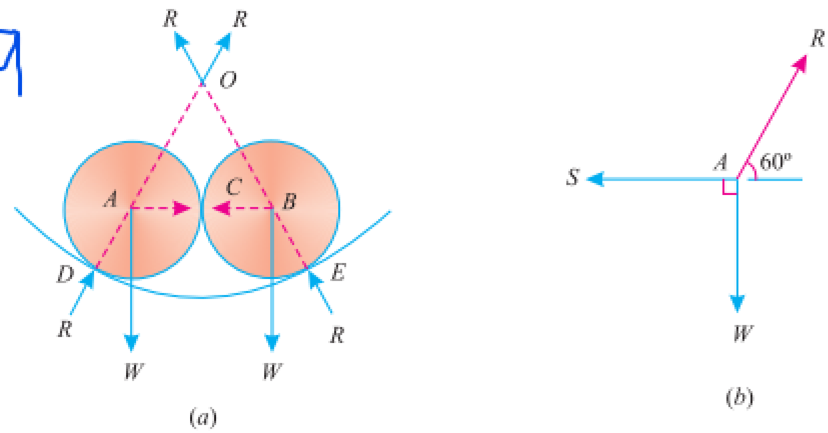

Fig. 5.11.

The two spheres with centres A and B, lying in equilibrium, in the cup with O as centre are shown in Fig. 5.11 (a). Let the two spheres touch each other at C and touch the cup at D and E respectively.

Let R = Reactions between the spheres and cup, and

S = Reaction between the two spheres at C.

From the geometry of the figure, we find that OD = 150 mm and AD = 50 mm. Therefore OA = 100 mm. Similarly OB = 100 mm. We also find that AB = 100 mm. Therefore OAB is an equilateral triangle. The system of forces at A is shown in Fig. 5.11 (b).

Applying Lami's equation at A,

$$\frac{R}{\sin 90°} = \frac{W}{\sin 120°} = \frac{S}{\sin 150°}$$

$$\frac{R}{1} = \frac{W}{\sin 60°} = \frac{S}{\sin 30°}$$

$$\therefore \quad R = \frac{S}{\sin 30°} = \frac{S}{0.5} = 2S$$

Hence the reaction between the cup and the sphere is double than that between the two spheres. **Ans.**