MIMLI DAS

Lecturer, Mechanical Engineering

ENGINEERING MECHANICS

CHAPTER 6: MOMENT OF INERTIA

Lecture 1:

6.1 Moments of inertia, Definition, Mathematical forces Unit.

Moment of Inertia (M.I) : The moment of inertia of an object is defined by the distribution of mass around an axis. It depends not only on the total mass of the object, but also on the square of the perpendicular distance from the axis to each element of mass. This means the moment of inertia increases rapidly as masses are distributed more distant from the axis.

Definition: The moment of a force (P) about a point, is the product of the force and perpendicular distance (x) between the point and the line of action of the force (i.e. P.x). This moment is also called first moment of force. If this moment is again multiplied by the perpendicular distance (x) between the point and the line of action of the force i.e. P.x (x) = Px^2, then this quantity is called moment of the moment of a force or second moment of force or moment of inertia.

Note: Sometimes, instead of force, area or mass of a figure or body is taken into consideration. Then the second moment is known as second moment of area or second moment of mass. But all such second moments are broadly termed as moment of inertia.

In this chapter, we will discuss the moment of inertia of plane areas only.

Moment of Inertia of a Plane Area: We consider a plane area, whose moment of inertia is required to be found out. We split up the whole area into a number of small elements.

Let $a_1, a_2, a_3, \ldots =$ Areas of small elements, and

$r_1, r_2, r_3, \ldots =$ Corresponding distances of the elements from the line about which the moment of inertia is required to be found out.

Now the moment of inertia of the area,

$$I = a_1 r_1^2 + a_2 r_2^2 + a_3 r_3^2 + \ldots$$
$$= \sum a\, r^2$$

Units: As a matter of fact, the units of moment of inertia of a plane area depend upon the units of the area and the length. e.g.,

1. If area is in $m^2$and the length is also in m, the moment of inertia is expressed in m^4.

2. If area in mm^2 and the length is also in mm, then moment of inertia is expressed in mm^4.

Methods for finding moment of inertia:

The moment of inertia of a plane area (or a body) may be found out by any one of the following two methods : 1. By Routh's rule 2. By Integration.

Note : The Routh's Rule is used for finding the moment of inertia of a plane area or a body of uniform thickness.

Moment of Inertia by Routh's Rule:

The Routh's Rule states, if a body is symmetrical about three mutually perpendicular axes, then the moment of inertia, about any one axis passing through its centre of gravity is given by:

$$I = \frac{A \text{ (or } M) \times S}{3} \quad \text{... (For a Square or Rectangular Lamina)}$$

$$I = \frac{A \text{ (or } M) \times S}{4} \quad \text{... (For a Circular or Elliptical Lamina)}$$

$$I = \frac{A \text{ (or } M) \times S}{5} \quad \text{... (For a Spherical Body)}$$

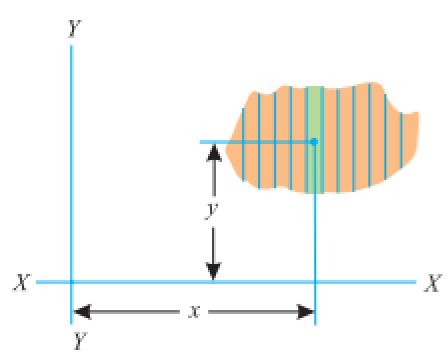
where A = Area of the plane area

M = Mass of the body, and

S = Sum of the squares of the two semi-axis, other than the axis, about which the moment of inertia is required to be found out.

Note : This method has only academic importance and is rarely used in the field of science and technology these days. The reason for the same is that it is equally convenient to use the method of integration for the moment of inertia of a body.

Moment of Inertia by Integration:



We consider a plane figure, whose moment of inertia is required to be found out about X-X axis and Y-Y axis as shown in figure above. Let us divide the whole area into a number of strips and consider one of these strips.

Let dA = Area of the strip ,

x and y = Distance of the centre of gravity of the strip on X-X axis and Y-Y axis respectively.
We know, the moment of inertia of the strip about Y-Y axis = dA . x^2

Now the moment of inertia of the whole area about Y-Y and X-X axis respectively may be found out by integrating above equation. i.e.,

$$\boldsymbol{I_{YY}} = \sum \mathbf{dA} \, . \, \boldsymbol{x^2}$$

Similarly, $$\boldsymbol{I_{XX}} = \sum \mathbf{dA} \, . \, \boldsymbol{y^2}$$