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Lecturer, Mechanical Engineering

ENGINEERING MECHANICS

CHAPTER 6: MOMENT OF INERTIA

Lecture 2:

6.2 M.I. of Plane figures like triangle, rectangles and circles problems.

Moment of Inertia of Rectangular Section:

We consider a rectangular section ABCD as shown in figure whose moment of inertia is required to be found out.

Let b= width of the section and d= depth of the section.

Now consider a strip *PQ* of thickness *dy* parallel to *X-X* axis and at a distance *y* from it as shown in the figure

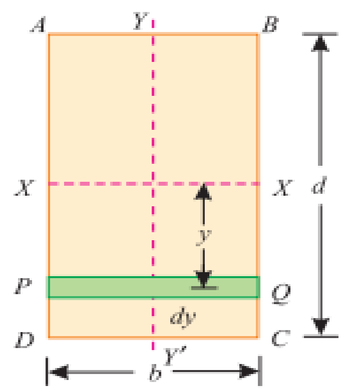
∴ Area of the strip

$= b.dy$

We know that moment of inertia of the strip about *X-X* axis,

$= \text{Area} \times y^2 = (b.\,dy)\,y^2 = b.\,y^2.\,dy$

Now *moment of inertia of the whole section may be found out by integrating the above equation for the whole length of the lamina *i.e.* from $-\frac{d}{2}$ to $+\frac{d}{2}$,



$$I_{xx} = \int_{-\frac{d}{2}}^{+\frac{d}{2}} b.y^2.dy = b \int_{-\frac{d}{2}}^{+\frac{d}{2}} y^2\,.\,dy$$

$$= b\left[\frac{y^3}{3}\right]_{-\frac{d}{2}}^{+\frac{d}{2}} = b\left[\frac{(d/2)^3}{3} - \frac{(-d/2)^3}{3}\right]$$

Therefore, $$I_{XX} = \frac{bd^3}{12}$$

Similarly, $$I_{YY} = \frac{db^3}{12}$$

Note: Cube is to be taken of the side, which is at right angles to the line of reference.

Example 7.1. *Find the moment of inertia of a rectangular section 30 mm wide and 40 mm deep about X-X axis and Y-Y axis.*

Solution. Given: Width of the section (b) = 30 mm and depth of the section (d) = 40 mm.

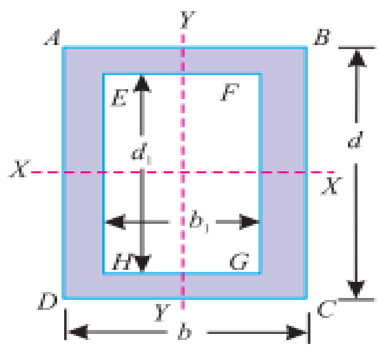
We know that moment of inertia of the section about an axis passing through its centre of gravity and parallel to X-X axis,

$$I_{XX} = \frac{bd^3}{12} = \frac{30 \times (40)^3}{12} = 160 \times 10^3 \text{ mm}^4 \quad \textbf{Ans.}$$

Similarly $$I_{YY} = \frac{db^3}{12} = \frac{40 \times (30)^3}{12} = 90 \times 10^3 \text{ mm}^4 \quad \textbf{Ans.}$$

Moment of Inertia of a Hollow Rectangular Section:

We consider a hollow rectangular section, in which ABCD is the main section and EFGH is the cut out section as shown in figure.



Let b = Breadth of the outer rectangle,

d = Depth of the outer rectangle and

b_1, d_1 = Corresponding values for the cut out rectangle.

We know that the moment of inertia, of the outer rectangle $ABCD$ about X-X axis

$$= \frac{bd^3}{12} \quad ...(i)$$

and moment of inertia of the cut out rectangle $EFGH$ about X-X axis

$$= \frac{b_1 d_1^3}{12} \quad ...(ii)$$

$\therefore$ M.I. of the hollow rectangular section about X-X axis,

I_{XX} = M.I. of rectangle $ABCD$ – M.I. of rectangle $EFGH$

$$= \frac{bd^3}{12} - \frac{b_1 d_1^3}{12}$$

Similarly, $$I_{yy} = \frac{db^3}{12} - \frac{d_1 b_1^3}{12}$$

Note: This relation holds good only if the centre of gravity of the main section as well as that of the cut out section coincide with each other.

Example 7.2. *Find the moment of inertia of a hollow rectangular section about its centre of gravity if the external dimensions are breadth 60 mm, depth 80 mm and internal dimensions are breadth 30 mm and depth 40 mm respectively.*

Solution. Given: External breadth (b) = 60 mm; External depth (d) = 80 mm ; Internal breadth (b_1) = 30 mm and internal depth (d_1) = 40 mm.

We know that moment of inertia of hollow rectangular section about an axis passing through its centre of gravity and parallel to X-X axis,

$$I_{XX} = \frac{bd^3}{12} - \frac{b_1 d_1^3}{12} = \frac{60\,(80)^3}{12} - \frac{30\,(40)^3}{12} = 2400 \times 10^3 \text{ mm}^4 \qquad \textbf{Ans.}$$

Similarly,

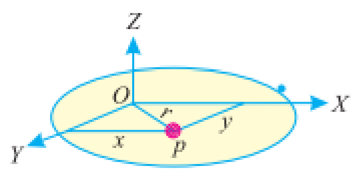
$$I_{YY} = \frac{db^3}{12} - \frac{d_1 b_1^3}{12} = \frac{80\,(60)^3}{12} - \frac{40\,(30)^3}{12} = 1350 \times 10^3 \text{ mm}^4 \qquad \textbf{Ans.}$$

Theorem of Perpendicular Axis:

It states that, " If I_{XX} and I_{YY} be the moments of inertia of a plane section about two perpendicular axis meeting at O, the moment of inertia I_{ZZ} about the axis Z-Z, perpendicular to the plane and passing through the intersection of X-X and Y-Y is given by:

$$I_{ZZ} = I_{XX} + I_{YY}$$

Proof: We consider a small lamina (P) of area *da* having co-ordinates as x and y along OX and OY , two mutually perpendicular axes on a plane section as shown in figure.



Now consider a plane OZ perpendicular to OX and OY. Let (r) be the distance of the lamina (P) from Z-Z axis such that $OP = r$.

From the geometry of the figure, we find that

$$r^2 = x^2 + y^2$$

We know that the moment of inertia of the lamina P about X-X axis,

$$I_{XX} = da.\, y^2 \qquad \ldots[\because I = \text{Area} \times (\text{Distance})^2]$$

Similarly, $$I_{YY} = da.\, x^2$$

and $$I_{ZZ} = da.\, r^2 = da\,(x^2 + y^2) \qquad \ldots(\because r^2 = x^2 + y^2)$$

$$= da.\, x^2 + da.\, y^2 = I_{YY} + I_{XX}$$