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Lecturer, Mechanical Engineering

ENGINEERING MECHANICS

CHAPTER 6: MOMENT OF INERTIA

Lecture 3:

Moment of Inertia of a Circular Section:

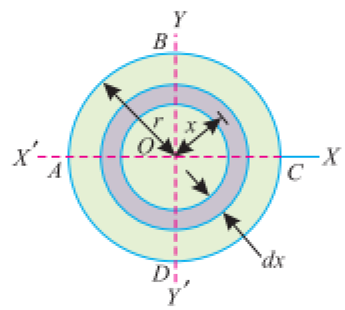
We consider a circle ABCD of radius (r) with centre O and X-X' and Y-Y' be two axes of reference through O as shown in figure.

Now consider an elementary ring of radius x and thickness dx. Therefore area of the ring,

$$da = 2\pi x.\, dx$$

and moment of inertia of ring, about X-X axis or Y-Y axis

$$= \text{Area} \times (\text{Distance})^2$$
$$= 2\pi x.\, dx \times x^2$$
$$= 2\pi x^3.\, dx$$



Now moment of inertia of the whole section, about the central axis, can be found out by integrating the above equation for the whole radius of the circle *i.e.*, from 0 to r.

$$\therefore \quad I_{ZZ} = \int_0^r 2\pi x^3 .\, dx = 2\pi \int_0^r x^3 .\, dx$$

$$I_{ZZ} = 2\pi\left[\frac{x^4}{4}\right]_0^r = \frac{\pi}{2}(r)^4 = \frac{\pi}{32}(d)^4 \qquad \ldots\left(\text{substituting } r = \frac{d}{2}\right)$$

We know from the Theorem of Perpendicular Axis that

$$I_{XX} + I_{YY} = I_{ZZ}$$

$$\therefore \qquad {}^*I_{XX} = I_{YY} = \frac{I_{ZZ}}{2} = \frac{1}{2} \times \frac{\pi}{32}(d)^4 = \frac{\pi}{64}(d)^4$$

Example 7.3. *Find the moment of inertia of a circular section of 50 mm diameter about an axis passing through its centre.*

Solution. Given: Diameter (d) = 50 mm

We know that moment of inertia of the circular section about an axis passing through its centre,

$$I_{XX} = \frac{\pi}{64}(d)^4 = \frac{\pi}{64} \times (50)^4 = 307 \times 10^3 \text{ mm}^4 \qquad \textbf{Ans.}$$

Moment of Inertia of a Hollow Circular Section:

We consider a hollow circular section as shown in figure whose moment of inertia is required to be found out.

Let D = Diameter of the main circle, and

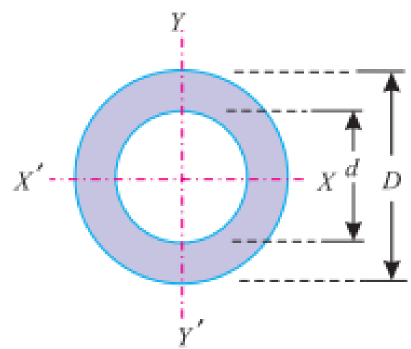
d = Diameter of the cut out circle.

We know that the moment of inertia of the main circle about X-X axis

$$= \frac{\pi}{64}(D)^4$$

and moment of inertia of the cut-out circle about X-X axis

$$= \frac{\pi}{64}(d)^4$$



$\therefore$ Moment of inertia of the hollow circular section about X-X axis,

I_{XX} = Moment of inertia of main circle – Moment of inertia of cut out circle,

$$= \frac{\pi}{64}(D)^4 - \frac{\pi}{64}(d)^4 = \frac{\pi}{64}(D^4 - d^4)$$

Similarly, $$I_{YY} = \frac{\pi}{64}(D^4 - d^4)$$

Note : This relation holds good only if the centre of the main circular section as well as that of the cut out circular section coincide with each other.

Example 7.4. *A hollow circular section has an external diameter of 80 mm and internal diameter of 60 mm. Find its moment of inertia about the horizontal axis passing through its centre.*

Solution. Given : External diameter (D) = 80 mm and internal diameter (d) = 60 mm.

We know that moment of inertia of the hollow circular section about the horizontal axis passing through its centre,

$$I_{XX} = \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}[(80)^4 - (60)^4] = 1374 \times 10^3 \text{ mm}^4 \quad \textbf{Ans.}$$

Theorem of Parallel Axis:

It states, If the moment of inertia of a plane area about an axis through its centre of gravity is denoted by I_G, then moment of inertia of the area about any other axis AB, parallel to the first, and at a distance h from the centre of gravity is given by:

$$I_{AB} = I_G + ah^2$$

Where, I_{AB}= Moment of inertia of the area about an axis AB,

I_G = Moment of Inertia of the area about its centre of gravity

a = Area of the section, and

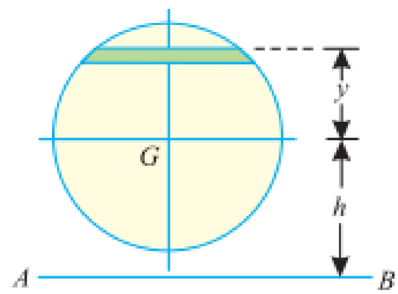
h = Distance between centre of gravity of the section and axis AB

Proof: We consider a strip of a circle, whose moment of inertia is required to be found out about a line AB as shown in figure.

Let δa = Area of the strip

y = Distance of the strip from the centre of gravity the section and

h = Distance between centre of gravity of the section and the axis AB.



We know that moment of inertia of the whole section about an axis passing through the centre of gravity of the section

$= \delta a . y^2$

and moment of inertia of the whole section about an axis passing through its centre of gravity,

$$I_G = \Sigma\, \delta a .\, y^2$$

∴ Moment of inertia of the section about the axis AB,

$$I_{AB} = \Sigma\, \delta a\, (h + y)^2 = \Sigma\, \delta a\, (h^2 + y^2 + 2\, h\, y)$$

$$= (\Sigma\, h^2 .\, \delta a) + (\Sigma\, y^2 .\, \delta a) + (\Sigma\, 2\, h\, y\, .\, \delta a)$$

$$= a\, h^2 + \; I_G + 0$$

It may be noted that $\Sigma\, h^2 .\, \delta a = a\, h^2$ and $\Sigma\, y^2 .\, \delta a = I_G$ [as per equation (*i*) above] and $\Sigma\, \delta a . y$ is the algebraic sum of moments of all the areas, about an axis through centre of gravity of the section and is equal to $a.\overline{y}$, where $\overline{y}$ is the distance between the section and the axis passing through the centre of gravity, which obviously is zero.