MIMLI DAS

Lecturer, Mechanical Engineering

ENGINEERING MECHANICS

CHAPTER 6: MOMENT OF INERTIA

Lecture 4:

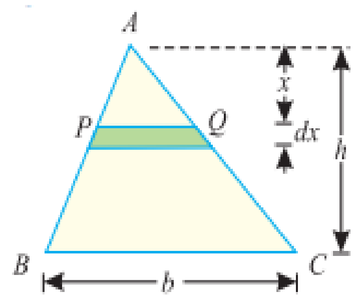
Moment of Inertia of a Triangular Section:

We consider a triangular section ABC whose moment of inertia is required to be found out.

Let b = Base of the triangular section and h = Height of the triangular section.

Now consider a small strip PQ of thickness dx at a distance of x from the vertex A as shown in Fig. 7.8. From the geometry of the figure, we find that the two triangles APQ and ABC are similar. Therefore

$$\frac{PQ}{BC} = \frac{x}{h} \quad \text{or} \quad PQ = \frac{BC.x}{h} = \frac{bx}{h}$$



We know that area of the strip PQ

$$= \frac{bx}{h} . dx$$

and moment of inertia of the strip about the base BC

$$= \text{Area} \times (\text{Distance})^2 = \frac{bx}{h} dx\,(h-x)^2 = \frac{bx}{h}(h-x)^2\,dx$$

Now moment of inertia of the whole triangular section may be found out by integrating the above equation for the whole height of the triangle *i.e.*, from 0 to h.

$$I_{BC} = \int_0^h \frac{b\,x}{h}(h-x)^2\;dx$$

$$= \frac{b}{h}\int_0^h x\,(h^2 + x^2 - 2\,h\,x)\;dx$$

$$= \frac{b}{h}\int_0^h (x\,h^2 + x^3 - 2\,hx^2)\;dx$$

$$= \frac{b}{h}\left[\frac{x^2 h^2}{2} + \frac{x^4}{4} - \frac{2hx^3}{3}\right]_0^h = \frac{bh^3}{12}$$

We know that distance between centre of gravity of the triangular section and base *BC*,

$$d = \frac{h}{3}$$

∴ Moment of inertia of the triangular section about an axis through its centre of gravity and parallel to *X-X* axis,

$$I_G = I_{BC} - ad^2 \qquad \ldots(\because I_{XX} = I_G + a\,h^2)$$

$$= \frac{bh^3}{12} - \left(\frac{bh}{2}\right)\left(\frac{h}{3}\right)^2 = \frac{bh^3}{36}$$

Notes : 1. The moment of inertia of section about an axis through its vertex and parallel to the base

$$= I_G + ad^2 = \frac{bh^3}{36} + \left(\frac{bh}{2}\right)\left(\frac{2h}{3}\right)^2 = \frac{9bh^3}{36} = \frac{bh^3}{4}$$

2. This relation holds good for any type of triangle.

Example. 7.5. *An isosceles triangular section ABC has base width 80 mm and height 60 mm. Determine the moment of inertia of the section about the centre of gravity of the section and the base BC.*

Solution. Given : Base width (b) = 80 mm and height (h) = 60 mm.

Moment of inertia about the centre of gravity of the section

We know that moment of inertia of triangular section about its centre of gravity,

$$I_G = \frac{bh^3}{36} = \frac{80 \times (60)^3}{36} = 480 \times 10^3 \text{ mm}^4$$

Moment of inertia about the base BC

We also know that moment of inertia of triangular section about the base *BC*,

$$I_{BC} = \frac{bh^3}{12} = \frac{80 \times (60)^3}{12} = 1440 \times 10^3 \text{ mm}^4$$

Exmple 7.6. *A hollow triangular section shown in Fig. 7.9 is symmetrical about its vertical axis.*

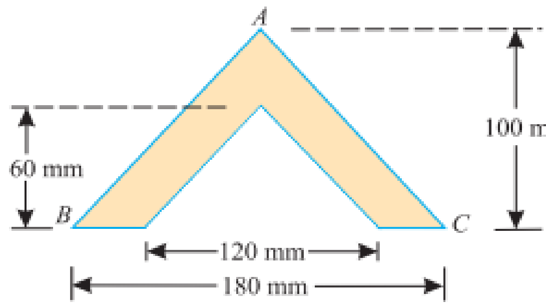


Fig. 7.9.

Find the moment of inertia of the section about the base BC.

Solution. Given : Base width of main triangle (B) = 180 mm; Base width of cut out triangle (b) = 120 mm; Height of main triangle (H) = 100 mm and height of cut out triangle (h) = 60 mm.

We know that moment of inertia of the triangular, section about the base BC,

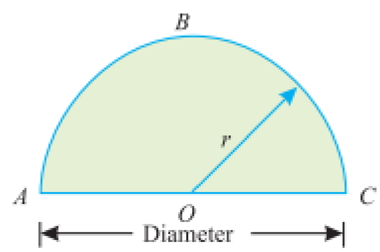
$$I_{BC} = \frac{BH^3}{12} - \frac{bh^3}{12} = \frac{180 \times (100)^3}{12} - \frac{120 \times (60)^3}{12} \text{ mm}^4$$

$$= (15 \times 10^6) - (2.16 \times 10^6) = 12.84 \times 10^6 \text{ mm}^4 \quad \textbf{Ans.}$$

Moment of Inertia of a Semi-Circular Section:

We consider a semicircular section ABC whose moment of inertia is required to be found out as shown in figure.

Let r = Radius of the semicircle.



We know that moment of inertia of the semicircular

section about the base AC is equal to half the moment of inertia of the circular section about AC.

Therefore moment of inertia of the semicircular section ABC about the base AC,

$$I_{AC} = \frac{1}{2} \times \frac{\pi}{64} \times (d)^4 = 0.393\, r^4$$

We also know that area of semicircular section,

$$a = \frac{1}{2} \times \pi r^2 \; \frac{\pi r^2}{2}$$

and distance between centre of gravity of the section and the base AC,

$$h = \frac{4r}{3\pi}$$

$\therefore$ Moment of inertia of the section through its centre of gravity and parallel to x-x axis,

$$I_G = I_{AC} - ah^2 = \left[\frac{\pi}{8} \times (r)^4\right] - \left[\frac{\pi r^2}{2}\left(\frac{4r}{3\pi}\right)^2\right]$$

$$= \left[\frac{\pi}{8} \times (r)^4\right] - \left[\frac{8}{9\pi} \times (r)^4\right] = 0.11\, r^4$$

Note. The moment of inertia about y-y axis will be the same as that about the base AC *i.e.*, $0.393\, r^4$.

Example 7.7. *Determine the moment of inertia of a semicircular section of 100 mm diameter about its centre of gravity and parallel to X-X and Y-Y axes.*

Solution. Given: Diameter of the section (d) = 100 mm or radius (r) = 50 mm

Moment of inertia of the section about its centre of gravity and parallel to X-X axis

We know that moment of inertia of the semicircular section about its centre of gravity and parallel to X-X axis,

$$I_{XX} = 0.11\, r^4 = 0.11 \times (50)^4 = 687.5 \times 10^3 \text{ mm}^4 \quad \textbf{Ans.}$$

Moment of inertia of the section about its centre of gravity and parallel to Y-Y axis.

We also know that moment of inertia of the semicircular section about its centre of gravity and parallel to Y-Y axis.

$$I_{YY} = 0.393\, r^4 = 0.393 \times (50)^4 = 2456 \times 10^3 \text{ mm}^4 \quad \textbf{Ans.}$$

Example 7.8. *A hollow semicircular section has its outer and inner diameter of 200 mm and 120 mm respectively as shown in Fig. 7.11.*

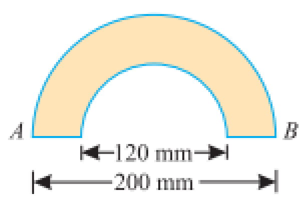


Fig. 7.11.

What is its moment of inertia about the base AB ?

Solution. Given: Outer diameter (D) = 200 mm or Outer Radius (R) = 100 mm and inner diameter (d) = 120 mm or inner radius (r) = 60 mm.

We know that moment of inertia of the hollow semicircular section about the base AB,

$$I_{AB} = 0.393\,(R^4 - r^4) = 0.393\,[(100)^4 - (60)^4] = 34.21 \times 10^6 \text{ mm}^4 \quad \textbf{Ans.}$$