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Lecturer, Mechanical Engineering

ENGINEERING MECHANICS

CHAPTER 8: MOTION

Lecture 1:

8.1 Linear and circular motion, Linear and angular velocities and acceleration, Units relation in between centrifugal force, Its uses in Engineering problems.

Linear Motion:

Linear motion, also called rectilinear motion is a motion on a straight line in one-dimension. When the object travels along a straight line it changes its position along with time. It is the most basic of all motion.

Example of linear motion - a car moving in a straight road.

Some terms required in this chapter:

1. Speed: The speed of a body may be defined as its rate of change of displacement with respect to its surroundings. The speed of a body is irrespective of its direction and is, thus, a scalar quantity.

2. Velocity: The velocity of a body may be defined as its rate of change of displacement, with respect to its surroundings, in a particular direction. As the velocity is always expressed in particular direction, therefore it is a vector quantity.

3. Acceleration: The acceleration of a body may be defined as the rate of change of its velocity. It is said to be positive, when the velocity of a body increases with time, and negative when the velocity decreases with time. The negative acceleration is also called retardation. In general, the term acceleration is used to denote the rate at which the velocity is changing. It may be uniform or variable.

4. Uniform acceleration: If a body moves in such a way that its velocity changes in equal magnitudes in equal intervals of time, it is said to be moving with a uniform acceleration.

5. Variable acceleration: If a body moves in such a way, that its velocity changes in unequal magnitudes in equal intervals of time, it is said to be moving with a variable acceleration.

6. Distance traversed: It is the total distance moved by a body. Mathematically, if body is moving with a uniform velocity (v), then in (t) seconds, the distance traversed

$$s = vt.$$

Motion under uniform acceleration:

We consider linear motion of a particle starting from O and moving along OX with a uniform acceleration as shown in figure. Let P be its position after t seconds.

Let u = Initial velocity,
v = Final velocity,
t = Time (in seconds) taken by the particle to change its velocity from u to v.
a = Uniform positive acceleration, and
s = Distance travelled in t seconds.

Since in t seconds, the velocity of the particle has increased steadily from (u) to (v) at the rate of a, therefore total increase in velocity

$$= a\,t$$

$$\therefore \quad v = u + a\,t \qquad \ldots(i)$$

and average velocity $= \left(\dfrac{u+v}{2}\right)$

We know that distance travelled by the particle,

$$s = \text{Average velocity} \times \text{Time}$$

$$= \left(\frac{u+v}{2}\right) \times t \qquad \ldots(ii)$$

Substituting the value of v from equation (i),

$$s = \left(\frac{u+u+at}{2}\right) \times t = ut + \frac{1}{2}at^2$$

From equation (i), (*i.e.* $v = u + at$) we find that

$$t = \frac{v-u}{a}$$

Now substituting this vlaue of t in equation (ii),

$$s = \left(\frac{u+v}{2}\right) \times \left(\frac{v-u}{a}\right) = \frac{v^2 - u^2}{2a}$$

or $\quad 2as = v^2 - u^2$

$$\therefore \quad v^2 = u^2 + 2as$$

Example 17.1. *A car starting from rest is accelerated at the rate of 0.4 m/s². Find the distance covered by the car in 20 seconds.*

Solution. Given : Initial velocity (u) = 0 (because, it starts from rest) ; Acceleration (a) = 0.4 m/s^2 and time taken (t) = 20 s

We know that the distance covered by the car,

$$s = ut + \frac{1}{2}at^2 = (0 \times 20) + \frac{1}{2} \times 0.4 \times (20)^2 \text{ m} = 80 \text{ m} \quad \textbf{Ans.}$$

Example 17.2. *A train travelling at 27 km.p.h is accelerated at the rate of 0.5 m/s². What is the distance travelled by the train in 12 seconds ?*

Solution. Given : Initial velocity (u) = 27 km.p.h. = 7.5 m/s ; Accceleration (a) = 0.5 m/s^2 and time taken (t) = 12 s.

We know that distance travelled by the train,

$$s = ut + \frac{1}{2}at^2 = (7.5 \times 12) + \frac{1}{2} \times 0.5 \times (12)^2 \text{ m}$$

$$= 90 + 36 = 126 \text{ m} \quad \textbf{Ans.}$$

Example 17.3. *A scooter starts from rest and moves with a constant acceleration of 1.2 m/s². Determine its velocity, after it has travelled for 60 meters.*

Solution. Given : Initial velocity (u) = 0 (because it starts from rest) Acceleration (a) = 1.2 m/s^2 and distance travelled (s) = 60 m.

Let v = Final velocity of the scooter.

We know that $v^2 = u^2 + 2as = (0)^2 + 2 \times 1.2 \times 60 = 144$

$$v = 12 \text{ m/s} = \frac{12 \times 3600}{1000} = 43.2 \text{ km.p.h.} \quad \textbf{Ans.}$$

Example 17.5 *A motor car takes 10 seconds to cover 30 meters and12 seconds to cover 42 meters. Find the uniform acceleration of the car and its velocity at the end of 15 seconds.*

Solution. Given : When $t = 10$ seconds, $s = 30$ m and when $t = 12$ seconds, $s = 42$ m.

Uniform acceleration

Let u = Initial velocity of the car, and

a = Uniform acceleration.

We know that the distance travelled by the car in 10 seconds,

$$30 = ut + \frac{1}{2}at^2 = u \times 10 + \frac{1}{2} \times a(10)^2 = 10u + 50\,a$$

Multiplying the above equation by 6,

$$180 = 60u + 300a \qquad ...(i)$$

Similarly, distance travelled by the car in 12 seconds,

$$42 = u \times 12 + \frac{1}{2} \times a(12)^2 = 12u + 72a$$

Mulitiplying the above equation by 5,

$$210 = 60u + 360a \qquad ...(ii)$$

Subtracting equation (*i*) from (*ii*),

$$30 = 60a \quad \text{or} \quad a = \frac{30}{60} = 0.5 \text{ m/s}^2 \quad \textbf{Ans.}$$

Velocity at the end of 15 seconds

Substituting the value of a in equation (*i*)

$$180 = 60u + (300 \times 0.5) = 60u + 150$$

$$\therefore \quad u = \frac{(180 - 150)}{60} = 0.5 \text{ m/s}$$

We know that the velocity of the car after 15 seconds,

$$v = u + at = 0.5 + (0.5 \times 15) = 8 \text{ m/s} \quad \textbf{Ans.}$$

Motion under variable acceleration:

We consider a body moving from O in the direction OX. Let P be its position at any instant as shown in figure.

Let s = Distance travelled by the body,

t = Time taken by the body, in seconds, to travel this distance

v = Velocity of the body, and

a = Acceleration of the body.

We know that the velocity of a body, is the rate of change of its position. Mathematically :

$$v = \frac{ds}{dt} \quad ...(i)$$

Similarly, acceleration of a body is the rate of change of its velocity. Mathematically :

$$a = \frac{d^2s}{dt^2} = \frac{dv}{dt} = v.\frac{dv}{ds} \quad ...(ii)$$

The velocity, acceleration and displacement of a body, from a mathematical expression, may be found out by either of the following two methods :

1. By differentiations and 2. By integration.