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ENGINEERING MECHANICS

CHAPTER 8: MOTION

Lecture 4:

Laws of motion: Following are the three laws of motion, which were enunciated by Newton, who is regarded as father of the Science.

1. **Newton's First Law of Motion** states, "Everybody continues in its state of rest or of uniform motion, in a straight line, unless it is acted upon by some external force." It is also called **Law of Inertia.**

2. **Newton's Second Law of Motion** states, "The rate of change of momentum is directly proportional to the impressed force, and takes place in the same direction, in which the force acts." This law enables us to measure a force, and establishes the fundamental equation of dynamics. It is also called the **Law of dynamics**.

3. **Newton's Third Law of Motion** states, "To every action, there is always an equal and opposite reaction."

Note: 1. A body at rest has a tendency to remain at rest. It is called inertia of rest.
2. A body in uniform motion in a straight line has a tendency to preserve its motion. It is called inertia of motion.

Force relation derived from Newton's Second law of motion:
We consider a body moving in a straight line. Let its velocity be changed while moving.

Let m = Mass of a body,

u = Initial velocity of the body,

v = Final velocity of the body,

a = Constant acceleration,

t = Time, in seconds required to change the velocity from u to v, and

F = Force required to change velocity from u to v in t seconds.

∴ Initial momentum $= mu$

and final momentum $= mv$

∴ Rate of change of momentum

$$= \frac{mv - mu}{t} = \frac{m(v-u)}{t} = ma \qquad \ldots\left[\because \frac{v-u}{t} = a\right]$$

According to Newton's Second Law of Motion, the rate of change of momentum is directly proportional to the the impressed force.

$$\therefore \qquad F \propto ma = kma$$

where k is a constant of proportionality.

For the sake of convenience, the unit of force adopted is such that it produces unit acceleration to a unit mass.

$$\therefore \qquad F = ma = \text{Mass} \times \text{Acceleration}.$$

Example 24.2. *A body has 50 kg mass on the earth. Find its weight (a) on the earth, where $g = 9.8\ m/s^2$; (b) on the moon, where $g = 1.7\ m/s^2$ and (c) on the sun, where $g = 270\ m/s^2$.*

Solution. Given: Mass of body (m) = 50 kg ; Acceleration due to gravity on earth (g_e) = 9.8 m/s^2;Acceleration due to gravity on moon (g_m) = 1.7 m/s^2 and acceleration due to gravity on sun (g_s) = 270 m/s^2.

(*a*) *Weight of the body on the earth*

We know that weight of the body on the earth

$$F_1 = mg_e = 50 \times 9.8 = 490 \text{ N} \qquad \textbf{Ans.}$$

(*b*) *Weight of the body on the moon*

We know that weight of the body on the moon,

$$F_2 = mg_m = 50 \times 1.7 = 85 \text{ N} \qquad \textbf{Ans.}$$

(*c*) *Weight of the body on the sun*

We also know that weight of the body on the sun,

$$F_3 = mg_s = 50 \times 270 = 13500 \text{ N} = 13.5 \text{ kN} \qquad \textbf{Ans.}$$

Example 24.3. *A body of mass 7.5 kg is moving with a velcoity of 1.2 m/s. If a force of 15 N is applied on the body, determine its velocity after 2 s.*

Solution. Given: Mass of body = 7.5 kg ; Velocity (u) = 1.2 m/s ; Force (F) = 15 N and time (t) = 2 s.

We know that acceleration of the body

$$a = \frac{F}{m} = \frac{15}{7.5} = 2 \text{ m/s}^2$$

∴ Velocity of the body after 2 seconds

$$v = u + at = 1.2 + (2 \times 2) = 5.2 \text{ m/s} \qquad \textbf{Ans.}$$

Example 24.[illegible]. *A vehicle, of mass 500 kg, is moving with a velocity of 25 m/s. A force of 200 N acts on it for 2 minutes. Find the velocity of the vehicle :*

(1) when the force acts in the direction of motion, and

(2) when the force acts in the opposite direction of the motion.

Solution. Given : Mass of vehicle (m) = 500 kg ; Initial velocity (u) = 25 m/s ; Force (F) = 200 N and time (t) = 2 min = 120 s

1. *Velocity of vehicle when the force acts in the dirction of motion*

We know that acceleration of the vehicle,

$$a = \frac{F}{m} = \frac{200}{500} = 0.4 \text{ m/s}^2$$

∴ Velocity of the vehicle after 120 seconds

$$v_1 = u + at = 25 + (0{\cdot}4 \times 120) = 73 \text{ m/s} \quad \textbf{Ans.}$$

2. *Velocity of the vehicle when the force acts in the opposite direction of motion.*

We know that velcoity of the vehicle in this case after 120 seconds, (when $a = -0.4$ m/s^2),

$$v_2 = u + at = 25 + (-0.4 \times 120) = -23 \text{ m/s} \quad \textbf{Ans.}$$

Minus sign means that the vehicle is moving in the reverse direction or in other words opposite to the direction in which the vehicle was moving before the force was made to act.

Example 24.10 *A body of mass 10 kg is moving over a smooth surface, whose equation of motion is given by the relation.*

$$s = 5t + 2t^2$$

where (s) is in metres and (t) in seconds. Find the magnitude of force responsible for the motion.

Solution. Given : Equation of motion : $s = 5t + 2t^2$

Differentiating both sides of the above equation with respect to t,

$$\frac{ds}{dt} = 5 + 4t$$

Again differentiating both sides of the above equation with respect to t,

$$\frac{d^2s}{dt^2} = 4 \quad \text{or} \quad \text{acceleration, } a = 4 \text{ m/s}^2.$$

∴ Force responsible for the motion,

$$F = ma = 10 \times 4 = 40 \text{ N} \quad \textbf{Ans.}$$

Circular motion or motion around a circular path as curvilinear motion:

Circular motion is a movement of an object along the circumference of a circle or rotation along a circular path. In circular motion, the distance between the body and a fixed point on the surface remains the same.

Examples of circular motion includes an artificial satellite orbiting the earth at a constant height, a stone which is tied to a rope and is being swung in circles, a car turning through a curve in race track.

Motion of an object moving in a curved path is called curvilinear motion. An object is said to be in a curvilinear motion if the direction of the velocity of its movement is variable and the path of the object is a curved line. Therefore motion along a circular path is a type of curvilinear motion.

Centripetal acceleration: The acceleration of a body traversing a circular path is called centripetal acceleration. Because velocity is a vector quantity (i.e, it has both magnitude and direction), when a body travels on a circular path, its direction constantly changes and thus its velocity changes, producing an acceleration. The acceleration is directed radially toward the centre of the circular path. The centripetal acceleration a_c has a magnitude equal to the square of the body's speed v along the curve divided by the distance r from the centre of the circle to the moving body. Mathematically, $a_c = v^2/r$.

We know v = ϖr, so putting this value in the above equation,

we have $a_c = \varpi^2 r^2/r = \varpi^2 r$

The formula is same for centrifugal acceleration also.

Centripetal acceleration has units of metre per second square $(\frac{m}{s^2})$. Centripetal means "towards the center" or "center seeking."

Centripetal force:

The force causing centripetal acceleration is called centripetal force and is always directed towards the centre of the circle. It acts along the radius of the circle at every point. It is the net force that acts on an object to keep it moving along a circular path.

Centrifugal force:

According to Newton's Third Law of Motion, the force, which acts opposite to the centripetal force, is known as centrifugal force. It may be noted that the centrifugal force always acts away from the centre of the path, or in other words, the centrifugal force always tends to throw the body away from the centre of circular path.

The centripetal or centrifugal force is written mathematically as

$F = m\, a_c = m\varpi^2 r$ (when ϖ is given)

$= \frac{mv^2}{r}$ (when v is given)

Example 28.1. *A body of mass 5 kg is moving in a circle of radius of 1·5 m with an angular velocity of 2 rad/s. Find the centrifugal force acting on the body.*

Solution. Given : Mass of body (m) = 5 kg ; Radius of circle (r) = 1·5 m and angular velocity of the body (ω) = 2 rad/s.

We know that centrifugal force acting on the body,

$$F = m\,\omega^2 r = 5 \times (2)^2 \times 1{\cdot}5 = 30 \text{ N} \quad \textbf{Ans.}$$

Example 28.2 *A stone of mass 1 kg is revolving in a circle of radius 1 m with a linear velocity of 10 m/s. What is the value of centrifugal force acting on the stone.*

Solution. Given : Mass of stone (m) = 1 kg ; Radius of circle (r) = 1 m and linear velocity of the stone (v) = 10 m/s.

We know that centrifugal force acting on the stone,

$$F = \frac{mv^2}{r} = \frac{1 \times (10)^2}{1} = 100 \text{ N} \qquad \textbf{Ans.}$$

Example 28.4. *A body of mass 0·5 kg tied to string is whirled in a vertical circle making 2 rev/s. If radius of the circle is 1·2 m, then find tensions in the string when the body is at top of the circle, and (ii) at the bottom of the circle.*

Solution. Given : Mass of body (m) = 0·5 kg ; Angular rotation of the body (N) = 2 rev/s and radius of circle (r) = 1·2 m.

(i) Tension in the string when the body is at the top of the circle

We know that angular velocity of the body,

$$\omega = 2\pi N = 2\pi \times 2 = 4\pi \text{ rad/s}$$

∴ Tension in the string when the body is at the top of the circle

$$T_1 = m\,\omega^2 r - mg = [0{\cdot}5 \times (4\pi)^2 \times 1{\cdot}2] - (0{\cdot}5 \times 9{\cdot}8) \text{ N}$$

$$= 94.7 - 4.9 = 89.8 \text{ N} \qquad \textbf{Ans.}$$

(ii) Tension in the string when the body is at the bottom of the circle

We know that tension in the string when the body is at the bottom of the circle

$$T_2 = m\,\omega^2 r + mg = [0{\cdot}5 \times (4\pi)^2 \times 1{\cdot}2] + (0{\cdot}5 \times 9{\cdot}8) \text{ N}$$

$$= 94{\cdot}7 + 4{\cdot}9 = 99{\cdot}6 \text{ N} \qquad \textbf{Ans.}$$

Example 28.5. *In a circus show, a motor cyclist is moving in a spherical cage of radius 3 m. The motor cycle and the rider together has mass of 750 kg. Find the least velocity, with which the motor cyclist must pass the highest-point on the cage, without losing contact inside the cage.*

Solution. Given : Radius of spherical cage (r) = 3 m and mass of motor cycle and rider (m) = 750 kg.

Let v = Least velocity of the motor cyclist.

We know that centrifugal force,

$$F = \frac{mv^2}{r} = \frac{750\,v^2}{3} = 250\,v^2$$

In order to maintain the contact with the highest point of the cage, the centrifugal force must be equal to the weight of the motor cycle and the rider. Therefore

$$250\,v^2 = mg = 750 \times 9{\cdot}8 = 7350$$

∴ $$v^2 = \frac{7350}{250} = 29{\cdot}4$$

or $$v = \sqrt{29{\cdot}4} = 5{\cdot}42 \text{ m/s} \qquad \textbf{Ans.}$$