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ENGINEERING MECHANICS

CHAPTER 8: MOTION

Lecture 5:

8.2 Angle of banking super elevation problems:

Super elevation: We know that the body, moving along a curved path, is subjected to the following forces : 1. Its own weight, and 2. Centrifugal force.

The resultant of these forces is inclined with the vertical as shown in Figure. It may be noted, that if curved path be made level, then the resultant force will be inclined at some angle, with the vertical, and thus the reactions on both the supports, of a vehicle, will not be equal, which will effect the equilibrium of the vehicle.

To counterbalance this effect and maintain equilibrium of the vehicle, the surface of the path is made perpendicular to the resultant by keeping the inner edge level and raising the outer edge of the roadway or (outer rail of the railway). The amount by which the outer edge of rail is raised is known as cant or superelevation.

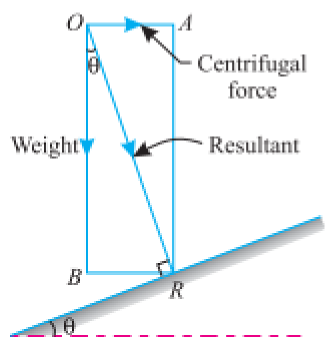


Figure: Super Elevation

Effect of superelevation in roadways: (Angle of banking)

In case of roadways, the outer edge is raised with respect to the inner edge of the road surface. The amount, by which the outer edge is raised is known as cant or superelevation. The process of providing superelevation is known as banking of the road.

The general practice, to define the superelevation, is to mention the angle of inclination of the road surface called angle of banking as shown in Figure.

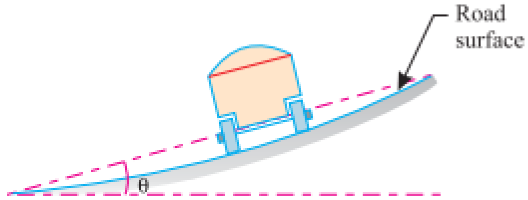


Figure: Superelevation in roadways

Here, a vehicle is moving on a roadway and along a curved path with a uniform velocity.
Let m = Mass of the body in tonnes,
r = Radius of circular path in m,
v = Velocity of the body in m/s, and
θ = Angle of the bank.
Whenever a body is moving along the circumference of a circle, it is subjected to the following forces :

1. Its own weight = mg
2. Centrifugal force = $\frac{mv^2}{r}$

From the geometry of the figure, we find that

$$\tan\theta = \frac{Centrifugal\ force}{Weight\ of\ the\ vehicle} = \frac{\frac{mv^2}{r}}{mg} = \frac{v^2}{gr}$$

$$\text{Therefore, } \boldsymbol{\theta = \tan^{-1}\left(\frac{v^2}{gr}\right)}$$

It may be noted from the above expression that the superelevation is independent of the mass of the body.

Example 28.8. *A circular automobile test track has a radius of 200 m. The track is so designed that when a car travels at a speed of 90 kilometres per hour, the force between the automobile and the track is normal to the surface of the track. Find the angle of the bank.*

Solution. Given : Radius of the track (r) = 200 m and speed of the car (v) = 90 km.p.h. = 25 m/s.

Let θ = Angle of the bank.

We know that $\tan\theta = \frac{v^2}{gr} = \frac{(25)^2}{9 \cdot 8 \times 200} = 0.3189$

$\theta = 17{\cdot}7°$ **Ans.**

Effect of superelevation in railways:

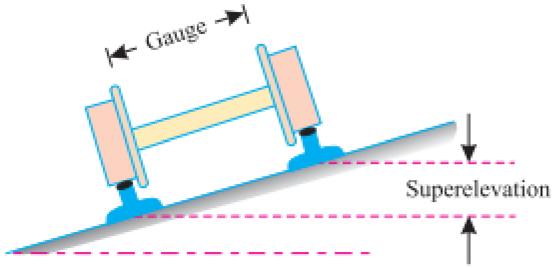


Figure: Superelevation in railways

In case of railways, the outer rail is raised with respect to the inner rail of the track. The amount by which the outer rail is raised is known as superelevation. The general practice, to define the superelevation, is to mention the difference of levels between the two rails as shown in Figure.

In such a case, the superelevation is given by the relation,

$$S = \frac{Gv^2}{gr},$$ where G is the gauge of the track.

The superelevation obtained, by this relation, is popularly known as equilibrium superelevation.

Example 28.9. *The distance between the rails of the track is 1·67 m. How much the outer rail be elevated for a curve of 500 m radius, in order that the resultant force may be normal at a speed of 45 km. p.h.*

Solution. Given : Gauge of the track (G) = 1·67 m ; Radius of the curve (r) = 500 m and speed (v) = 45 km.p.h. = 12·5 m/s

We know that the superelevation,

$$S = \frac{Gv^2}{gr} = \frac{1{\cdot}67 \times (12{\cdot}5)^2}{9{\cdot}8 \times 500} = 0{\cdot}0533 \text{ m} = 53{\cdot}3 \text{ mm} \quad \textbf{Ans.}$$

Example 28.10 *Find the superelevation to be provided on a 1·67 m gauge curved track of 1000 m radius, if the speeds of the trains are as follows :*

(a) 15 trains at a speed of 50 km.p.h.
(b) 10 trains at a speed of 60 km.p.h.
(c) 5 trains at a speed of 70 km.p.h.
(d) 2 trains at a speed of 80 km.p.h.

Solution. Given : Gauge of the track (G) = 1·67 m and radius of curved track (r) = 1000 m

We know that the equilibrium speed,

$$v = \frac{(15 \times 50) + (10 \times 60) + (5 \times 70) + (2 \times 80)}{15 + 10 + 5 + 2} = \frac{1860}{32} \text{ km.p.h.}$$

$$= 58{\cdot}125 \text{ km.p.h.} = 16{\cdot}15 \text{ m/s}$$

and superelevation,

$$S = \frac{Gv^2}{gr} = \frac{1{\cdot}67\,(16{\cdot}15)^2}{9{\cdot}8 \times 1000} = 0{\cdot}044 \text{ m} = 44 \text{ mm} \qquad \textbf{Ans.}$$

8.3 Bodies moving on a level circular path, skidding, overturning.

Reactions of a vehicle moving along a level circular path:

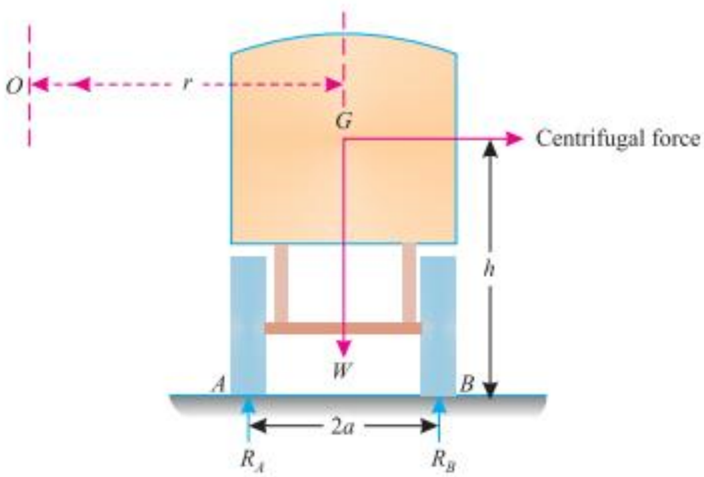


Figure: Reactions of a vehicle

We consider a vehicle moving on a level circular path, with O as centre as shown in figure.

Let m = Mass of the vehicle,
v = Velocity of the vehicle,
r = Radius of the circular path,
h = Height of the centre of gravity of the vehicle from the ground level,
R_A = Reaction at A (*i.e.* inner wheel)
R_B = Reaction at B, (*i.e.* outer wheel) and
$2a$ = Distance between the reactions at A and B

We know that the centrifugal force, acting through the centre of gravity of the vehicle,

$$P_c = \frac{mv^2}{r}$$

We also know that the weight of the vehicle (equal to mg) acting downwards through its centre of gravity will exert a reaction equal to $\frac{mg}{2}$ on A and B (because of symmetry).

Taking moments about A and equating the same,

$$R_B \times 2a = (mg \times a) + \left(\frac{mv^2}{r} \times h\right) = mag\left(1 + \frac{v^2 h}{gra}\right)$$

$$\therefore \quad R_B = \frac{mg}{2}\left(1 + \frac{v^2 h}{gra}\right)$$

Similarly, by taking moments about B and equating the same, we get

$$R_A = \frac{mg}{2}\left(1 - \frac{v^2 h}{gra}\right)$$

Example 28.11. *A vehicle of mass 1200 kg. is to turn a level circular curve of radius 100 metres with a velocity of 30 km.p.h. The height of its c.g. above the road level is 1 metre and the distance between the centre lines of the wheels is 1·5 metre. Find the reactions of the wheels.*

Solution. Given : Mass of the vehicle (m) = 1200 kg. = 1·2 t ; Radius of the curve (r) = 100 m ; Velocity of the vehicle (v) = 30 km.p.h. = 8·33 m/s ; Height of the c.g. of the vehicle from the road level (h) = 1 m and distance between the centre lines of the wheel ($2a$) = 1·5 m or a = 0·75 m

Reaction at the inner wheel

We know that reaction at the inner wheel,

$$R_A = \frac{mg}{2}\left(1 - \frac{v^2 h}{gra}\right) = \frac{1{\cdot}2 \times 9{\cdot}8}{2}\left(1 - \frac{(8{\cdot}33)^2 \times 1}{9{\cdot}8 \times 100 \times 0{\cdot}75}\right) \text{ kN}$$

= 5.325 kN **Ans.**

Reaction at the outer wheel

We also know that reaction at the outer wheel,

$$R_B = \frac{mg}{2}\left(1 + \frac{v^2 h}{gra}\right) = \frac{1{\cdot}2 \times 9{\cdot}8}{2}\left(1 + \frac{(8{\cdot}33)^2 \times 1}{9{\cdot}8 \times 100 \times 0{\cdot}75}\right) \text{ kN}$$

= 6·435 kN **Ans.**

Equilibrium of a vehicle moving along a level circular path:

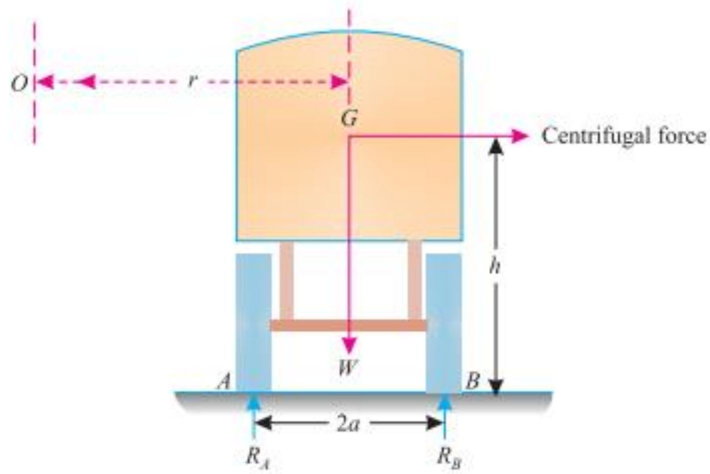
The main idea, of banking the road or providing the superelevation in the railway lines, is to distribute the load of the vehicle equally on both the wheels. But, if the roads are not banked, then a vehicle may also have to face the following mishappenings. This will also happen, when the vehicle moves with a velocity more than the permissible velocity.

1. The vehicle may overturn, or

2. The vehicle may skid away.

Now we shall discuss the maximum velocity of a vehicle, so that it may remain in an equilibrium state on a circular track. Or in other words, we shall discuss the maximum velocity to avoid both the above mishappenings, one by one.

Maximum velocity to avoid overturning of a vehicle moving along a level circular path:



We consider a vehicle moving on a level circular path, with O as centre as shown in figure.

Let m = Mass of the vehicle,

v = Velocity of the vehicle,

r = Radius of the circular path,

h = Height of the centre of gravity of the vehicle from the ground level,

R_A = Reaction at A (*i.e.* inner wheel)

R_B = Reaction at B, (*i.e.* outer wheel) and

$2a$ = Distance between the reactions at A and B

Here, we know that reaction at A is

$$R_A = \frac{mg}{2}\left(1 - \frac{v^2 h}{gra}\right)$$

and reaction at B,

$$R_B = \frac{mg}{2}\left(1 + \frac{v^2 h}{gra}\right)$$

A little consideration will show, that the reaction at B, (*i.e.* R_B) can never be negative. But the reaction A may be negative, if the value of $\frac{v^2 h}{gra}$ becomes more than unity (*i.e.* 1). This is only possible if the value of v is increased (because all the other things are constant). When this condition reaches, the vehicle will overturn at the wheel B. Therefore in order to avoid overturning,

$$\frac{v^2 h}{gra} < 1 \quad \text{or} \quad v < \sqrt{\frac{gra}{h}}$$

It is thus obvious, that if the velocity of the vehicle is less than that obtained from the above equation, the vehicle will not overturn. But if the velocity is more, the vehicle is bound to overturn. Therefore in order to avoid overturning the maximum velocity,

$$v_{max} = \sqrt{\frac{gra}{h}}$$

The maximum velocity of the vehicle to avoid overturning is independent of its mass.

Example 28.12. *A vehicle, weighing 1 tonnes is to turn on a circular curve of 40 m radius. The height of its centre of gravity above the road level is 75 cm and the distance between the centre lines of the wheel is 120 cm. Find the speed, at which the vehicle should be run, in order to avoid overturning.*

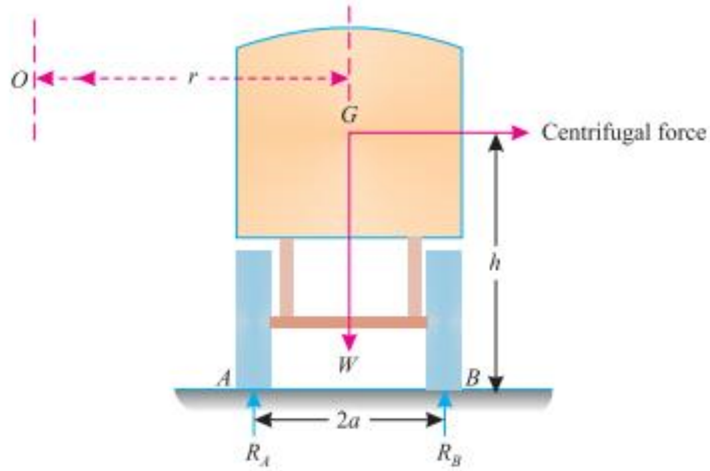
Solution. Given : Weight of vehicle = 1 t ; Radius of the curve (r) = 40 m ; Height of the centre of gravity of the vehicle from road level (h) = 75 cm = 0·75 m and distance between centre lines of the wheels ($2a$) = 120 cm = 1·2 m or a = 0·6 m

We know that maximum speed at which the vehicle should run, in order to avoid overturning,

$$v_{max} = \sqrt{\frac{gra}{h}} = \sqrt{\frac{9.8 \times 40 \times 0.6}{0.75}} = 17.7 \text{ m/s}$$

$$= 63.72 \text{ km.p.h.} \quad \textbf{Ans.}$$

Maximum velocity to avoid skidding away of a vehicle moving along a level circular path:



We consider a vehicle moving on a level circular path, with O as centre as shown in figure.

Let
m = Mass of the vehicle
v = Velocity of the vehicle
r = Radius of the circular path
μ = Coefficient of friction between the wheels of the vehicle and the ground.

We know that the centrifugal force, which tends to skid away the vehicle,

$$P_c = \frac{mv^2}{r}$$

and the force of friction between the wheels of the vehicle and ground

$$= \mu R_A + \mu R_B = \mu (R_A + R_B) = \mu mg$$

The skidding away of the vehicle can only be avoided, if the force of friction (between wheels of the vehicle and the ground) is more than the centrifugal force. Therefore in order to avoid skidding away,

$$\frac{mv^2}{r} < \mu mg \qquad \text{or} \qquad v < \sqrt{\mu gr}$$

It is thus obvious, that if the velocity of the vehicle is less than that obtained from the above equation, the vehicle will not skid away. But, if the velocity is more, the vehicle is bound to skid away. Therefore in order to avoid skidding the maximum velocity,

$$v_{max} = \sqrt{\mu gr}$$

The maximum velocity of the vehicle to avoid skidding is independent of its mass.

Example 28.13. *A car is travelling on a level track of radius 50 m. Find the maximum speed, at which he can travel on the curved track, if the coefficient of friction between the tyres and track is 0.45. Take $g = 9{\cdot}8\ m/s^2$.*

Solution. Given : Radius of level track (r) = 50 m ; Coefficient of friction (μ) = 0·45 and $g = 9{\cdot}8\ \text{m/s}^2$.

We know that maximum speed at which the car can travel,

$$v_{max} = \sqrt{\mu g r} = \sqrt{0{\cdot}45 \times 9{\cdot}8 \times 50} = 14{\cdot}85 \text{ m/s}$$

$$= 53.5 \text{ km.p.h.} \quad \textbf{Ans.}$$

Example 28.14. *A cyclist, riding at 5 m/s has to turn a corner. What is least radius of the curve, he has to describe, if the coefficient of friction between the tyres and the road be 0·25 ?*

Solution. Given : Velocity of the cycle (v) = 5 m/s and coefficient of friction (μ) = 0·25

Let r = Radius of the curve in metres.

We know that velocity of the cyclist (v),

$$5 = \sqrt{\mu g r} = \sqrt{0{\cdot}25 \times 9{\cdot}8 \times r} = \sqrt{2{\cdot}45\, r}$$

$$25 = 2{\cdot}45\, r \qquad \text{...(Squaring both sides)}$$

$$\therefore \quad r = \frac{25}{2{\cdot}45} = 10{\cdot}2 \text{ m} \quad \textbf{Ans.}$$

OBJECTIVE TYPE QUESTIONS

1. Which of the following statements is correct ?
   (*a*) When a body moves along a circular path with a uniform velocity, there will be a tangential acceleration.
   (*b*) When a body moves along a circular path with a uniform acceleration, there will be a tangential acceleration.
   (*c*) When a body moves along a circular path with a uniform velocity, there will be no tangential acceleration.
   (*d*) When a body moves along a circular path with a uniform acceleration, there will be no tangential acceleration.

2. The slope on the road surface generally provided on the curves is known as
   (*a*) angle of friction (*b*) angle of repose
   (*c*) angle of banking (*d*) none of the above

3. On a curved railway track, the amount by which the outer rail is raised, is known as superelevation.

(a) Yes (b) No

4. The superelevation is given by the relation

(a) $\frac{Gv}{gr}$ (b) $\frac{Gv^2}{gr}$ (c) $\frac{Gv}{gr^2}$ (d) $\frac{Gv^2}{gr^2}$

where G = Gauge of the track

v = Velocity of the vehicle, and

r = Radius of the circular path.

5. When a body is moving along a circular path, the centrifugal force tends to overturn the body. The chances of overturning can be decreased by decreasing the

(a) weight of the vehicle

(b) speed of the vehicle

(c) height of c.g.of the vehicle from the road level.

(d) all of the above.

6. The maximum velocity of a vehicle, in order to avoid skidding on a level circular path is

(a) μgr (b) $\frac{1}{2}\mu gr$ (c) $\sqrt{\mu gr}$ (d) $\frac{1}{2}\sqrt{\mu gr}$

ANSWERS

1. (c)	2. (c)	3. (a)	4. (b)	5. (b)	6. (c)